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On-line identification simulation of forgetting methods to track time varying parameters using alternative covariance matrix

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Abstract: The paper compares abilities of forgetting methods to track time varying parameters of two different simulated models with different types of excitation. The observed parameters in simulations are the integral sum of the Euclidean norm of a deviation of the parameter estimates from their true values and a selected band prediction error count. As supplementary information we observe the eigenvalues of the covariance matrix. In the paper we used modified method of Regularized Exponential Forgetting with Alternative Covariance Matrix (REFACM or REZAKM) along with Directional Forgetting (DF or SZ) and three standard regularized methods.

Keywords: online identification, time varying parameters, covariance matrix, forgetting

1. INTRODUCTION

This paper is devoted to online identification methods and their practical application possibilities along with adaptive control; while monitoring long-run operation of time variant dynamic systems. Emphasis is set on long-run operation and therefore the working mechanism with non-informative data. The process of algorithm realization is elaborated as well. Online identification methods are explored, where non-informative data which could possibly destabilize numerical computation of the identified system parameters, is weighted by the chosen method to ensure „forgetting". The contribution of this paper lies in two newly created algorithms and their modifications for online identification; based on the technique utilizing an alternative covariance matrix. All algorithms are validated by simulations in Matlab Simulink software environment. Finally, the results obtained through the simulation algorithms mentioned in the article are compared to other commonly used algorithms.

2. PROBLEM STATEMENT

The Let as consider a stochastic system on which observations are made at discrete time instants k = 1, 2... . A directly manipulated input u_k and an indirectly affected output y_k (booth possibly multivariate) can be distinguished in the data pair $d_k = (u_k, y_k)$. The collection of all data observed on the system up to time t is denoted by $D_t = (d_1, d_2, ... d_t)$. The dependence of a new pair of data (u_k, y_k) on previous observations D_{k-1} can be described by a conditional probability density function (p.d.f) with the following structure

$$p(y_k, u_k | D_{k-1}, \theta_k) = p(y_k | u_k, D_{k-1}, \theta_k) p(u_k | D_{k-1}) \qquad (1)$$

Incomplete knowledge of the system behavior is expressed through a vector of unknown, time varying parameters $\theta_k \in \theta$. Note that the input generator described by the second term does not depend on these parameters directly, it is expected to utilize only prior information and information contained in observed data. The first term actually characterizes the system.

3. REF AND SLZ TECHNIQUE

Suppose that no explicit model of parameter changes is known. Yet, we can quantify our prior information (and possibly information taken from data already available) by introducing an alternative probability density function (p.d.f.) $p^*(\theta_{k+1}/D_k)$. The problem is then to construct (p.d.f.) $p(\theta_{k+1}/D_k)$ based on two hypotheses described by the p.d.f. $p(\theta_k/D_k)$ (the case of no parameter changes) and the alternative p.d.f. $p^*(\theta_{k+1}/D_k)$ (the case of worst expected changes). For simplicity in this section we use the notation $p_0(\theta)$, $p_1(\theta)$ a $p^*(\theta)$ for the posterior, alternative and resulting p.d.f.'s, respectively. In Kulhavý and Kraus (1996), formulated the task of choosing p^* given p_0 and p_1 as a Bayesian decision making problem. In the next we will make a short review of their solutions, let

$$p_{\hat{\theta},P} = \frac{1}{\sqrt{2\Pi}} |P|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} (\theta - \hat{\theta})' P^{-1} (\theta - \hat{\theta}) \right) \qquad (2)$$

where $\hat{\theta}$ and P denote the mean and covariance of a particular p.d.f. then the following solutions were shown:

EF:

$$\hat{\theta}^* = \hat{\theta}_0, P^{*-1} = \lambda P_0^{-1} + (1 - \lambda) P_1^{-1} \qquad (3)$$

LF:

$$\hat{\theta}^* = \hat{\theta}_0, P^* = \lambda P_0 + (1-\lambda)P_1 \qquad (4)$$

Let's consider the model of system with time varying parameters θ_k, see Kulhavý and Kraus (1996). In order to be able to track parameter variations we complement the standard recursive last square (RLS) algorithm by exponential or linear forgetting according to (3) or (4) respectively. In addition the alternative mean is set equal to posterior mean $\hat{\theta}^{alt}_{k+1|k} = \hat{\theta}_{k|k}$ and for simplicity the alternative covariance is set equal to the prior covariance $P^{alt}_{k+1|k} = P_{1,0} = Q$. With this choice we can use general forgetting algorithm with the following choice of forgetting operator

$$F\{P_{k|k}, Q\} = \left[\lambda \quad P^{-1}_{k|k} + (1-\lambda)Q^{-1}\right]^{-1} \qquad (5)$$

which construct harmonic mean for REF (or REZ in tabs) and

$$F\{P_{k|k}, Q\} = \lambda \quad P_{k|k} + (1-\lambda)Q \qquad (6)$$

which construct arithmetic mean for SLF (or SLZ in tabs). In both cases the prior covariance matrix Q isn't forgotten and is repetitively taking into account in every step k see in Schmitz et al. (2003).

4. AUGMENTING REF AND SLF WITH ACM

The involved SLF and REF augmentation considers addition and keeping the initial information in the Alternative Covariance Matrix (ACM or AKM) form. The augmentation is based on the modified Dyadic reduction algorithm, where instead of adding a-priori covariance matrix Q, ACM is computed at each step. ACM is stabilizing the evolution of matrix P(0) after the recursive update. This operation is necessary for the SLF and REF algorithms to be augmented by the stabilization component in the ACM form. The aforementioned stabilization component prevents the destabilization of the original algorithms at long running employments; when slow time changes are to be expected in the observed parameters in relation to the sampling period. The modified algorithms have been named as follows: the modified and ACM augmented REF algorithm is to be called REFACM (or REZAKM in tabs), the modified SLF algorithm augmented by ACM will be named SLFACM (or SLZAKM in tabs).

5. SIMULATIONAL ALGORITHM VERIFICATION METHODOLOGY

Two different models were created for the verification of the properties of the introduced algorithms in the observation of time variant parameters of dynamic systems.. These two models (model no. 1. and no. 2.) have a different approach to input excitation (input signal generator A and B). Algorithm quality has been compared through the use of DF, which is considered to be standard in the field. The Praly Forgetting (PF or PZ in tabs) algorithm featured in the work Praly (1993) has been also used, using the weighted covariance matrix P. All algorithms were subject to the same test with the identical length using the two featured models.

All results were graphically evaluated, and analyzed in a table where algorithm quality has been shown numerically through parameters IS and PE.

5.1 Description of model no. 1 and no. 2

In the case of model 1, a second order model is considered with external disturbance $v_{(t)}$ according to:

$$y_k = \sum_{i=1}^{2} a_i y_{k-i} + \sum_{i=0}^{2} b_i u_{k-i} + \sum_{i=0}^{2} b_i u_{k-i} + e_t, e_k \approx N(0,\sigma^2) \qquad (7)$$

The values of constant parameters are given by: a_2 = -0.9, b_0 = 0.5, b_1 = -0.25, b_2 = 0.1, d_1 = 0.8, d_2 = 0.2 and σ = 0.1.

The time variant parameter has been chosen as $a_{(1)}$ = 0.98, which has been kept constant half of the n simulation steps, then at time t = n / 2 changed its value to $a_{(1)}$ = − 0.98. The outside disturbance has been simulated as a square signal periodically changing its value from +1 to −1 each hundred simulation steps. The identification has been made difficult mainly by the rarely occurring disturbances, which contained minimal information about the parameter $d_{(i)}$.

For the needs of the simulation, two input signal generators have been assumed:

- Input signal generator A: discrete white noise generator
- Input signal generator B: the input signal has been generated using the following equation: $u^*_k = 0.8u^*_{k-1} + 0.2u_k$, where u^*_k is normally distributed white noise and u^*_{k-1} is the previous input value.

For model no. 2. only one change has been realized in comparison to model no. 1. This has been carried out by altering the time variant parameter $a_{(1,k)}$ = 0.98 cos(2π k/250). In this case, two different input generators were considered as well:

- Input signal generator A: discrete white noise generator
- Input signal generator B: the input signal has been generated similarly to model no 1., where $u_{(k)}$ has been only chosen from the interval $u_{(k)}$ ~ (0.5, 1).

6. VERIFICATION – MATLAB SIMULINK

From the previously mentioned algorithms DF has been chosen along with the three regularized methods: REF, SLF and PF. For the simulation verification a set of S-Function libraries has been created along with a common universal user interface. This interface (Figure 1. and Figure 2.) allows the user to select input data, simulated model and the observed algorithm. Output of the discussed simulations is a graphical representation of the observed parameters along with a data file containing the results for the following analysis. Integral sum (IS) of the Euclidian norm of parameter error and prediction error PE has been shown, which is the amount exceeded by the interval ±3σ2. The simulation experiments will be marked by the character pair XY, where X is the number of the utilized model (no. 1 or no. 2) and Y represents the generator utilized (A, respectively B).

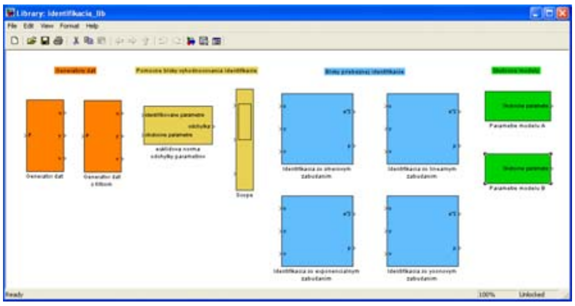

Fig. 1. User interface with selection of blocks

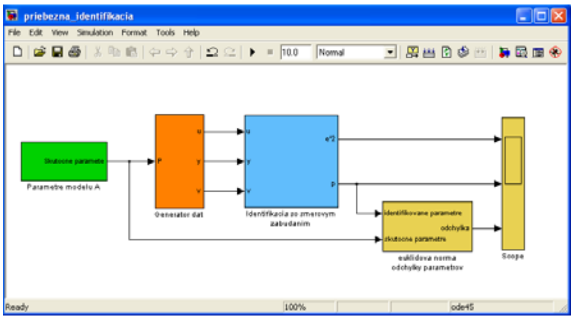

Fig. 2. Simulation block scheme in Simulink

7. EVALUATION OF SIMULATION RESULTS

This section introduces all results in a table form. For the detailed description of algorithm behavior during the simulations with different lengths, simulations lasting n = 1200, 6000, 12 000 and 120 000 have been evaluated as featured in Tables (1) to (4). It is clear from Table (3); that using simulation length n = 12 000 steps the artefacts of long lasting runs are already appearing. The result is the confirmation of REFACM algorithm quality in comparison to REF, which in the case 1A achieved better results then REF. Excellent results are achieved by the algorithm PF also. The least satisfactory performance is provided by the algorithm SLFACM. The data featured in Table (4) fully confirm the previous considerations of the REFACM algorithm quality. It is clear that using ACM as if a constraint has been enforced on parameter trending, which also implies the improvement of IS parameters in comparison of the results achieved by REF. The convergence of the REF covariance matrix is faster and finite in contrast to REFACM, where the convergence is slower and also the addition of excited ACM cannot be finite. The achieved simulation results and REFACM algorithm behaviour at 1200 and 120 000 simulation steps, show that as the running length increases the quality improves in contrast to REF. In Table (5) we have shown influence of weighting factor λ to quality of REFACM algorithms. Best results in simulations we observers in settings λ=0.8.

1 200 krokov	1A	**1B**	2A	**2B**
SZ	IS=409,7 PE=151	IS=390,7 PE=70	IS=689,6 PE=326	IS=805,7 PE=143
SLZ	IS=213,7 PE=24	IS=129,0 PE=13	IS=208,2 PE=50	IS=440,1 PE=24
REZ	IS=68,5 PE=16	IS=72,1 PE=11	IS=168,7 PE=38	IS=258,2 PE=26
PZ	IS=89,3 PE=23	IS=112,2 PE=14	IS=242,0 PE=38	IS=478,4 PE=37
SLZAKM	IS=780,2 PE=200	IS=779,6 PE=141	IS=1022,7 PE=110	IS=1012,2 PE=251
REZAKM	IS=103,9 PE=29	IS=101,1 PE=36	IS=339,6 PE=110	IS=474,1 PE=65

Tab. 1. Simulation length 1200 steps

6 000 krokov	1A	**1B**	2A	**2B**
SZ	IS=356,6 PE=130	IS=957,3 PE=131	IS=3499,4 PE=1723	IS=4016,0 PE=717
SLZ	IS=153,4 PE=18	IS=208,6 PE=12	IS=928,5 PE=210	IS=2683,3 PE=131
REZ	IS=118,3 PE=18	IS=120,3 PE=12	IS=740,9 PE=174	IS=1336,7 PE=103
PZ	IS=125,9 PE=23	IS=114,4 PE=13	IS=921,9 PE=197	IS=1807,2 PE=173
SLZAKM	IS=595,9 PE=247	IS=1062,3 PE=98	IS=4817,1 PE=2282	IS=7111,0 PE=1405
REZAKM	IS=127,4 PE=27	IS=177,8 PE=34	IS=963,8 PE=239	IS=1985,1 PE=166

Tab. 2. Simulation length 6000 steps

12 000 krokov	1A	**1B**	2A	**2B**
SZ	IS=645,7 PE=253	IS=1604,6 PE=214	IS=6701,0 PE=3892	IS=7474,5 PE=1488
SLZ	IS=219,2 PE=18	IS=257,5 PE=13	IS=1533,4 PE=363	IS=3589,9 PE=179
REZ	IS=168,9 PE=13	IS=253,9 PE=14	IS=1457,2 PE=351	IS=2525,3 PE=188
PZ	IS=103,5 PE=14	IS=165,4 PE=15	IS=1760,8 PE=379	IS=3210,8 PE=355
SLZAKM	IS=721,4 PE=174	IS=1365,4 PE=82	IS=9640,0 PE=5105	IS=12951 PE=2793
REZAKM	IS=162,9 PE=21	IS=296,3 PE=31	IS=1844,9 PE=654	IS=4507,3 PE=439

Tab. 3. Simulation length 12 000 steps

120 000 krokov	REZ	**REZAKM**
1A	IS=1035,4 PE=17	IS=880,8 PE=51
1B	IS=3419,5 PE=12	IS=2963,7 PE=29

Tab. 4. Simulation length 120 000 steps

REZAKM	$\lambda = 0.8$	$\lambda = 0.5$	$\lambda = 0.2$
1A	IS=127,4 PE=27	IS=164,2 PE=34	IS=182,8 PE=48
1B	IS=177,8 PE=34	IS=298,9 PE=50	IS=496,9 PE=94
2A	IS=963,6 PE=239	IS=870,1 PE=289	IS=1031,6 PE=327
2B	IS=1985,1 PE=166	IS=1520,0 PE=145	IS=1732,8 PE=167

Tab. 5. Influence of weighing factor λ

8. CONCLUSTION

The simulation verification tests featured in the previous section, evaluated in the Matlab Simulink environment confirm that the quality of the tested algorithms is diverse. From the viewpoint of our interest, that is the long running simulations, the best results are achieved by the REFACM algorithm – the contribution of this paper. The quality of REFACM in comparison with the other algorithms confirms the advantages of using ACM given the specific conditions featured in this work.

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