Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation, and Mathematics

PROCEEDINGS

of the 18$^{\mathrm{th}}$ International Conference on Process Control

Hotel Titris, Tatranská Lomnica, Slovakia, June 14 – 17, 2011

ISBN 978-80-227-3517-9

http://www.kirp.chtf.stuba.sk/pc11

Editors: M. Fikar and M. Kvasnica

PI/PID Controller Design for FOPDT Plants Based on the Modulus Optimum Criterion

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Abstract: We present the PI/PID controller settings for the first order systems with dead time, based on the modulus optimum criterion. The settings provide fast closed-loop response to changes of the reference input. Unlike most other tuning methods, the parameters are obtained without approximation of the delay term, so they remain valid for long dead time. Besides the performance indices, quality of the settings is also evaluated by the stability margin. Although optimal values of the parameters are valid for the reference tracking problem, a compensation of the disturbance lag that preserves the stability margin is proposed for the disturbance rejection problem.

1. INTRODUCTION

Many industrial processes are modelled by the stable first-order plus dead time (FOPDT) transfer function:

$$S(s) = \frac{K}{Ts+1} e^{-\tau s} \tag{1}$$

where K is the system gain, $T > 0$ is the time constant and τ is the dead time parameter. The model (1) allows simple experimental identification from the step response, which can be in most cases easily measured. Simple methods based on coincidence in one or more points and more complex methods suitable for noisy data are described in (Åström and Hägglund 1995) and (Kiong et al. 1999).

For tuning PID controllers based on the model (1) many approaches exist, see e.g. (Åström and Hägglund 1995) for description of the most important methods. A comprehensive survey of known formulas is available in (O´Dwyer 2003). Early methods were derived from empirical requirements on the step response, such as one-quarter decay ratio (Cohen and Coon 1953), (Ziegler and Nichols 1942), step-response overshoot (Chien at al. 1952) or from integral criterions in time domain with approximation of the dead-time dynamics (Lopez et al. 1967), (Wang et al. 1995). These methods, however, usually work well only for a rather limited range of the ratio τ / T .

Among methods for the model of type (1) without approximation of the delay term the design with given gain and phase margins (Ho et al. 1995) and LQR design (Kiong et al. 1999) should be mentioned. Alternative ways for systems with long time delay include internal model control (Rivera et al. 1986), Smith predictor and λ-tuning (Åström and Hägglund 1995). These approaches, however, require implementation of delay in the control system.

The method for setting up PI controller parameters based on cancellation of the factor $(Ts+1)$ was proposed in (Haalman 1965). In this method the dead-time dynamics is manipulated without approximation. Good reference tracking performance is achieved, but on the other hand, poor results may be observed for rejection of load disturbances (Åström and Hägglund 1995). In a similar way it is possible to compensate dynamics of the second order plus dead time system by a PID controller.

In this paper we utilize Haalman's idea of pole compensation for designing optimal PI and PID controller parameters for the model (1). The pole compensation fixes the value of one parameter of the controller. We adjust the remaining controller parameters to meet analytic design criteria. We show that in this case especially the modulus optimum criterion leads to a simple choice of the parameters and to a control loop with very good practical properties. In this case, derivative term of the controller increases both the performance and the stability margin. The results presented here appeared in full context in the journal paper (Cvejn 2009), where also the settings based on the minimum ISE criterion were analyzed.

The modulus optimum criterion introduced in (Oldenbourg and Sartorius 1956) requires that the amplitude of the closed-loop frequency response is close to one for low frequencies. If the closed-loop frequency response is decreasing, this condition is analogous to the requirement that the frequencies in the reference input are passed in the broadest possible range. Such a behaviour is desirable for the reference tracking cases, because then the closed-loop system is able to respond quickly to changes of the reference input.

Let us write the closed-loop frequency response in the form

$$T(\omega) = \frac{L(i\omega)}{1+L(i\omega)} = \frac{1}{1+1/L(i\omega)} \tag{2}$$

where $L(s)$ is the open-loop transfer function in Laplace transform. If $L(s)$ contains a pole in the origin, which is necessary to achieve asymptotically zero regulation error, for

$\omega \to 0$ holds $L(i\omega) \to \infty$ and thus $\lim_{\omega \to 0} |T(\omega)|^2 = 1$. Therefore, it is possible to write $|T(\omega)|^2$ as

$$|T(\omega)|^2 = 1 + H(\omega) \tag{3}$$

where $H(\omega) \in C_\infty$. Maximal flatness of the closed-loop frequency-response modulus is then equivalent to the requirement that

$$n_0\left(H(\omega)\right) \to \max \tag{4}$$

where $n_0\left(H(\omega)\right)$ denotes the index of the first nonzero coefficient in the Taylor expansion of $H(\omega)$.

Besides performance objectives, the design has to respect stability requirements. As the stability margin we consider the distance of the open-loop Nyquist plot from the critical point $[-1, 0]$, i.e. the value

$$\gamma = \inf_{\omega \in (0,\infty)} \{1 + L(i\omega)\}, \quad \gamma \in [0, 1]. \tag{5}$$

The reciprocal value of γ is known as the sensitivity function. In general case it is recommended that the sensitivity is in the range from 1.3 to 2 (Åström and Hägglund 1995).

2. THE CONTROLLER DESIGN

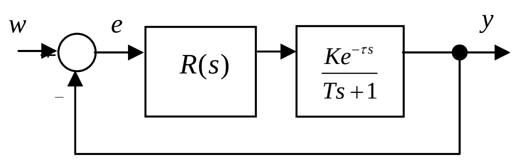

Fig. 1. Control scheme for reference tracking

At first, consider the PI controller case. Consider the reference tracking control problem in Fig. 1. If we compensate the factor $(Ts+1)$ by the PI controller

$$R(s) = K_C\left(1 + \frac{1}{T_I s}\right) \tag{6}$$

where

$$K_C = \frac{\kappa T}{K\tau}, \quad T_I = T \tag{7}$$

the open-loop transfer function is

$$L(s) = \frac{\kappa}{\tau s} e^{-\tau s} \tag{8}$$

where κ is a tuning parameter. The corresponding frequency response can be written as

$$L(\xi) = \frac{\kappa}{i\xi} e^{-i\xi} = \frac{\kappa}{\xi} e^{-i(\xi + \pi/2)} = -\kappa\left(\frac{\sin\xi}{\xi} + i\frac{\cos\xi}{\xi}\right) \tag{9}$$

where $\xi = \tau\omega$ is normalized frequency.

The corresponding Nyquist plot is dependent only on a single parameter κ, which can be adjusted so that sufficient stability margin is guaranteed and performance objectives are fulfilled.

In the case of serial PID controller we can put analogously

$$\begin{aligned} R(s) &= K_C\left(1 + \frac{1}{T_I s}\right)\left(1 + T_D s\right) = \\ &= \kappa \frac{T}{\tau K}\left(1 + \frac{1}{Ts}\right)\left(1 + T_D s\right) \end{aligned} \tag{10}$$

and we easily obtain the corresponding open-loop transfer function

$$L(s) = \kappa\left(\frac{T_D}{\tau} + \frac{1}{\tau s}\right) e^{-\tau s} \tag{11}$$

and the frequency response in the form

$$L(\xi) = \kappa\left(\delta - i\frac{1}{\xi}\right) e^{-i\xi} \tag{12}$$

where $\xi = \tau\omega$ is normalized frequency and $\delta = T_D / \tau$.

Proposition 1. The modulus-optimum settings for the PID controller (10) are:

$$\delta^* = 1/3 \text{ and } \kappa^* = 3/4. \tag{13}$$

and for the PI controller (6):

$$\kappa^* = 1/2. \tag{14}$$

Proof: The closed-loop frequency response (2) square modulus is

$$|T(\xi)|^2 = \frac{1}{1 + \frac{1}{|L|^2}\left(1 + 2\operatorname{Re} L\right)} = \frac{1}{1 + Q(\xi)}. \tag{15}$$

It can be easily verified (Cvejn 2009) that if

$$1 + H(\xi) = \frac{1}{1 + Q(\xi)} \tag{16}$$

where $Q(\xi) \in C_\infty$, it holds

$$n_0\left(H(\xi)\right) = n_0\left(Q(\xi)\right). \tag{17}$$

Since

$$|L(\xi)|^2 = \kappa^2\left(\delta^2 + \frac{1}{\xi^2}\right) \tag{18}$$

it is

$$n_0\left(Q(\xi)\right) = n_0\left(1 + 2\operatorname{Re} L(\xi)\right) + 2 = n_0\left(G(\xi)\right) + 1 \tag{19}$$

where

$$G(\xi) = \xi\left(1 + 2\operatorname{Re} L(\xi)\right). \tag{20}$$

To achieve maximal $n_0\left(Q(\xi)\right)$ and thus maximal $n_0\left(H(\xi)\right)$ we require that the derivatives of $G(\xi)$ are zero for $\xi \to 0$ up to maximal order. After substitution, it is easily found that

$$\begin{aligned} \lim_{\xi\to 0}\frac{dG}{d\xi} &= 1+2\kappa(\delta-1) \\ \lim_{\xi\to 0}\frac{d^3G}{d\xi^3} &= 2\kappa(1-3\delta) \\ \lim_{\xi\to 0}\frac{d^5G}{d\xi^5} &= 2\kappa(5\delta-1) \end{aligned} \qquad (21)$$

$$\text{and} \quad \lim_{\xi\to 0}\frac{d^{(k)}G}{d\xi^{(k)}} = 0,\ k=0,2,4,\ldots \qquad (22)$$

Putting (21) equal to zero yields $\delta^* = 1/3$ and $\kappa^* = 3/4$. For PI controller, where $\delta = 0$, it follows that the optimal setting is $\kappa^* = 1/2$. □

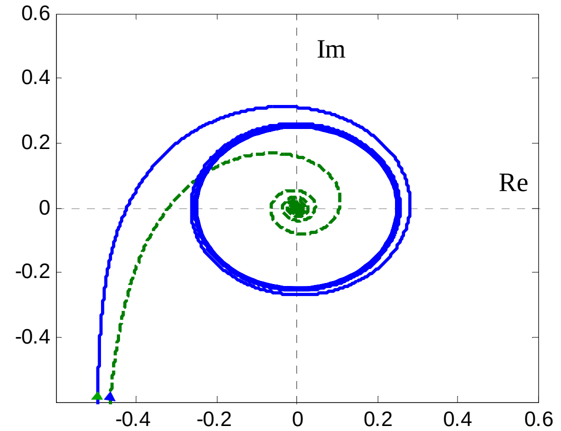

Fig. 2. Open-loop Nyquist plots of proposed settings (solid line – PID controller, dashed line – PI controller)

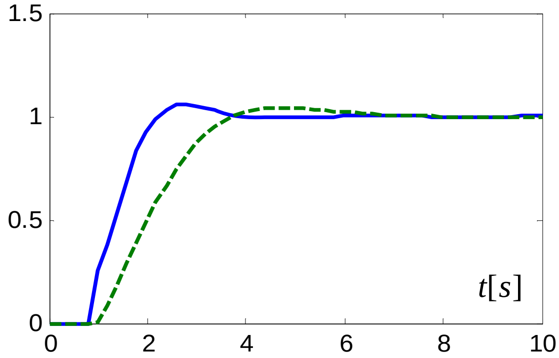

Fig. 3. Step responses (solid line – PID controller, dashed line – PI controller)

Figures 2 and 3 show the Nyquist plots and corresponding step responses for the proposed settings (the ideal open-loop transfer function (11) with $\tau = 1s$ is considered). Obtained settings obviously have very good quality for most practical purposes – the time response is fast and nearly not oscillating. The overshoot is of about 6 % in the case of PID controller. Figure 4 shows the corresponding dependence of $|T(\xi)|$ on ξ.

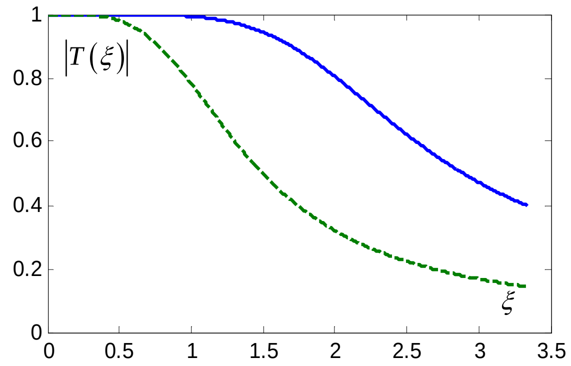

Fig. 4. Dependence $\zeta \to |T(\xi)|$ (solid line – PID controller, dashed line – PI controller)

The resulting parameters of PI, serial PID and the parallel PID controller

$$R(s) = K_C\left(1+\frac{1}{T_I s}+T_D s\right) \qquad (23)$$

are summarized in Tab. 1.

Controller	K_C	T_I	T_D
PI	$\frac{1}{2}\frac{T}{K\tau}$	T	-
PID (serial)	$\frac{3}{4}\frac{T}{K\tau}$	T	$\frac{1}{3}\tau$
PID (parallel)	$\frac{1}{4K}\left(1+\frac{3T}{\tau}\right)$	$T+\frac{\tau}{3}$	$\frac{\tau}{3+\tau/T}$

Tab. 1. PI/PID controller settings for reference tracking

For the ultimate normalized frequency, where $\arg L(\xi_c) = -\pi$, we easily obtain the equation

$$\xi_c = \pi - \mathrm{arctg}\left(\frac{1}{\delta\xi_c}\right) \qquad (24)$$

which can be solved iteratively. Denote ψ the angle between negative real axis and the Nyquist plot of $L(\xi)$ at the ultimate frequency $\xi = \xi_c$. Geometrical shape of the curve (Fig. 2) enables to construct a lower bound of the stability margin γ using the angle ψ :

$$\gamma \ge \underline{\gamma} = \left(1-\frac{1}{\alpha}\right)\sin\psi \qquad (25)$$

where α is the amplitude margin. $\underline{\gamma} \approx 0.54$ was obtained for PID controller and $\underline{\gamma} = 0.57$ for PI controller.

3. THE DISTURBANCE REJECTION PROBLEM

In most practical cases the reference input is held constant, but the system is excited by external disturbances. We consider that the disturbance influences output through the FOPDT transfer function (see Fig. 5).

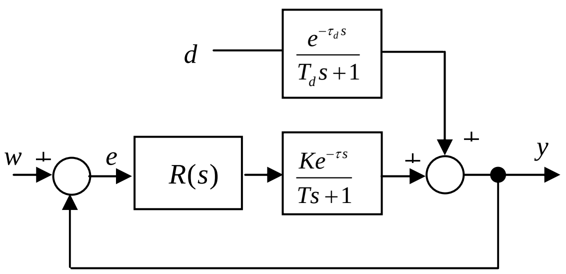

Fig. 5. Control scheme for disturbance rejection

It is well known that good tracking performance does not imply efficient disturbance rejection (Åström and Hägglund 1995). The closed-loop transfer function between *d* and *y* is

$$S_d(s) = \frac{e^{-\tau_d s}}{T_d s + 1} \frac{1}{1 + L(s)} \tag{26}$$

and thus the factor $1/\left(T_d s + 1\right)$ will be present in the response regardless of the controller settings, unless it is compensated by a closed-loop zero.

Since the rise time in the optimal configuration including dead time is not shorter than about 2τ in all the configurations, if $T_d \le 2\tau$, total dynamics is not affected much adversely by the term $1/\left(T_d s + 1\right)$ in the input. On the other hand, if $T_d \gg 2\tau$, the factor $1/\left(T_d s + 1\right)$ can slow down the response significantly.

The term on the right in (26) corresponds to the transfer function of the regulation error at the reference tracking problem and thus the optimal disturbance rejection problem is analogous to the problem of optimal tracking reference signal with L-transform

$$W(s) = \frac{e^{-\tau_d s}}{T_d s + 1} \frac{1}{s}. \tag{27}$$

Therefore, one way how to achieve good performance is to sufficiently decrease T_d, while keeping the other parts of the closed-loop transfer function unchanged. If d is not measured, both these objectives probably cannot be fulfilled. Below a compensation that reduces T_d to T_d' and simultaneously approximately preserves the stability margin is proposed. It is possible to assume that in this case the performance will not be much degraded.

If we choose the controller in the form

$$R_d(s) = \frac{T_d}{T_d'} \frac{T_d' s + 1}{T_d s + 1} R(s) \tag{28}$$

where $R(s)$ is the controller tuned for the reference tracking and $T_d' < T_d$, the closed-loop transfer function is

$$\begin{aligned} S_d(s) &= \frac{e^{-\tau_d s}}{T_d s + 1} \frac{1}{1 + \frac{T_d}{T_d'} \frac{T_d' s + 1}{T_d s + 1} L(s)} = \\ &= \frac{T_d'}{T_d} \frac{e^{-\tau_d s}}{T_d' s + 1} \frac{1}{\frac{T_d'}{T_d} \frac{T_d s + 1}{T_d' s + 1} + L(s)}. \end{aligned} \tag{29}$$

However, such a reduction of T_d at the same time decreases the stability margin. Denote $L_d(\xi)$ the open-loop frequency response if the controller (28) is used. If we assume that $\tau / \left(T_d' \xi\right) \ll 1$ and $T_d \ge T_d'$, holds

$$L_d(\xi) = \frac{T_d}{T_d'} \frac{i T_d' \xi / \tau + 1}{i T_d \xi / \tau + 1} L(\xi) \approx \left(1 - i \frac{1}{\xi r_d}\right) L(\xi) \tag{30}$$

where

$$r_d = \frac{1}{\tau} \left(\frac{1}{T_d'} - \frac{1}{T_d} \right)^{-1}. \tag{32}$$

We determine the parameter r_d from the condition

$$\frac{\underline{\gamma} - \underline{\gamma}_d}{\underline{\gamma}} = h \tag{33}$$

where $\underline{\gamma}$, $\underline{\gamma}_d$ are the lower estimates of the stability margin given by formula (25) for the original controller and the modified controller (28), respectively, and h is a sufficiently small chosen constant. Equation (33) is solved iteratively, see (Cvejn 2009) for complete explanation. T_d' is then obtained from

$$T_d' = \left(\frac{1}{r_d \tau} + \frac{1}{T_d} \right)^{-1}. \tag{34}$$

The value h should be chosen so that the stability margin be approximately preserved, but since small h leads to large T_d', a compromise has to be looked for. A good choice seems to lie in the range $h \in \left[0.05, 0.08\right]$. The results corresponding to $h = 1/16$ are $r_d = 5.92$ for PI controller and $r_d = 3.91$ for PID controller. Note that for $T_d / \tau \to 0$ we obtain $R_d(s) \to R(s)$ and the controller is the same as for the reference tracking problem.

Figure 6 shows the open-loop Nyquist plots after the compensation for PID controller, for $T_d \to \infty$, $T_d = 5\tau$, $T_d = \tau$ and $T_d = 0$ (i.e. without compensation).

A disadvantage of the proposed compensation is that additional term $(T_d's+1)/(T_d s+1)$ is needed. However, if $T_d = T$ (this is also the case when the disturbance influences the system input), the factor $1/(Ts+1)$ of the system need not be directly compensated. Instead, the PID controller takes the form

$$R_d(s)=\frac{T}{T_d'}\frac{T_d's+1}{Ts+1}\frac{\kappa T}{\tau K}\left(1+\frac{1}{Ts}\right)\left(T_D s+1\right)= =\frac{\kappa T}{\tau K}\left(1+\frac{1}{T_d{}'s}\right)\left(T_D s+1\right) \qquad (35)$$

which is similar to (10). Note that the integral term of the controller, which is needed to achieve zero regulation error, here plays the additional role of compensating the disturbance lag.

The corresponding settings of PI and PID controllers for input disturbance rejection, $h=1/16$, are summarized in Tab. 2.

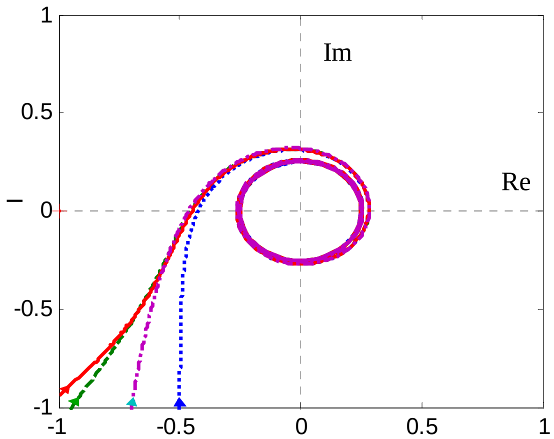

Fig. 6. Open-loop Nyquist plots after compensation, PID controller for $T_d \to \infty$ (solid line), $T_d = 5\tau$ (dashed), $T_d = \tau$ (dash-dotted) and $T_d = 0$ (dotted)

Ctrl.	K_C	T_I	T_D
PI	$\frac{1}{2}\frac{T}{K\tau}$	$\left(\frac{1}{5.9\tau}+\frac{1}{T}\right)^{-1}$	-
PID serial	$\frac{3}{4}\frac{T}{K\tau}$	$\left(\frac{1}{3.9\tau}+\frac{1}{T}\right)^{-1}$	$\frac{1}{3}\tau$
PID parallel	$\frac{1}{4K}\left(1+3.26\frac{T}{\tau}\right)$	$\left(\frac{1}{3.9\tau}+\frac{1}{T}\right)^{-1}+\frac{1}{3}\tau$	$\left(\frac{3.26}{\tau}+\frac{1}{T}\right)^{-1}$

Tab. 2. PI/PID controller settings for input disturbance rejection

4. EXPERIMENTAL COMPARISONS

At first, we consider the reference tracking problem, where the reference signal is not known in advance. Although many of PID tuning formulas for the model (1) are available, most of them are applicable only for a limited range of the ratio $\theta=\tau/T$. Usually, it is required that $\theta \ge 0.1$ and $\theta \le 1$ or $\theta \le 2$. The minimum ISE, IAE and ITAE tuning rules in (Wang et al. 1995), where the recommended range of θ is $\theta \in [0.05,\,6]$, are among the exceptions. Note that the modulus-optimum (MO) settings we propose admit any positive value of θ.

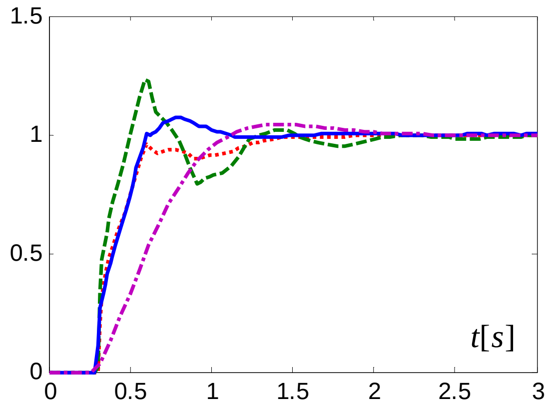

Fig. 7. $F_1(s)$ reference tracking, step response. Settings: MO-PID (solid line), MO-PI (dash-dotted), Chien (dashed), Wang (dotted)

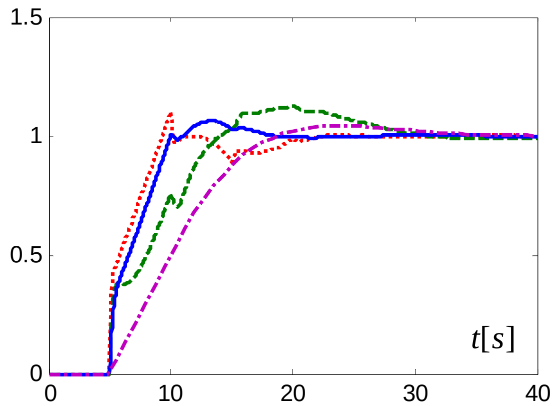

Fig. 8. $F_2(s)$ reference tracking, step response. Settings: MO-PID (solid line), MO-PI (dash-dotted), Chien (dashed), Wang (dotted)

Figures 7 and 8 show the reference signal step responses for the plants

$$F_1(s)=\frac{1}{s+1}e^{-0.3s},\quad F_2(s)=\frac{1}{s+1}e^{-5s} \qquad (36)$$

and parallel (ideal) PID controller tuned by using Wang IAE formulas (Wang et al. 1995), well known formulas by Chien et al. (Chien et al. 1952) for 20% step-response overshoot and

MO settings of PI and PID controller. Chien settings for large θ result in too slow response, which could be expected, since recommended range of θ is $\theta \in (0.11, 1)$. We also tested well known Ziegler-Nichols formula (Ziegler and Nichols 1942), which for $F_1(s)$ give a rather oscillating response with about 75% overshoot and for $F_2(s)$ slow and overdamped response.

For the disturbance rejection problem we consider that the disturbance influences the system input, i.e. $T_d = T$. Figures 9 an 10 show the load disturbance step responses for the plants $F_1(s)$ and $F_2(s)$ and parallel PID controller tuned according to the minimum IAE formulas in (Lopez et al. 1967), disturbance rejection formulas with 20% overshoot in (Chien et al. 1952) and MO-tuned PI and PID controllers with the input disturbance compensation. Obviously, Lopez and Chien formulas, recommended for $\theta \in [0.1, 1]$, are not suitable for large θ. Ziegler-Nichols settings give responses very similar to Chien settings, in both the cases. In all the cases the proposed settings give very satisfactory results.

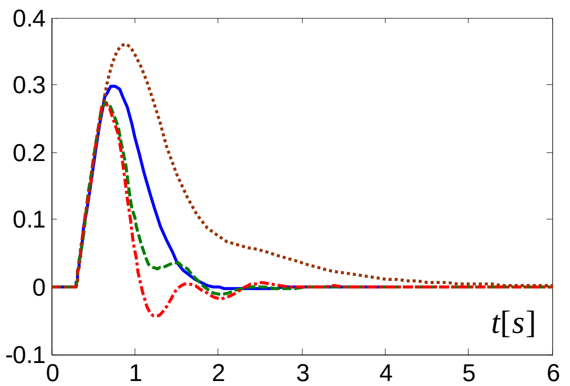

Fig. 9. $F_1(s)$ load disturbance, step response. Settings: MO (solid line), Chien (dashed), Lopez (dash-dotted), Haalman (PI) (dotted)

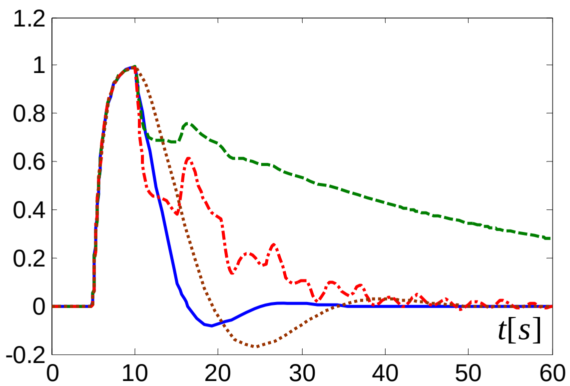

Fig. 10. $F_2(s)$ load disturbance, step response. Settings: MO (solid line), Chien (dashed), Lopez (dash-dotted), Haalman (PI) (dotted)

ACKNOWLEDGEMENTS

The work has been supported under the Czech research project MSM 0021627505.

REFERENCES

Åström, K.J., Hägglund, T. (1995). *PID Controllers: Theory, Design, and Tuning. 2nd edition*. Instrument Society of America (ISA).

Chien, K.L., Hrones, J.A., Reswick, J.B. (1952). On the automatic control of generalised passive systems. *Transactions of the ASME*, 74, 175-185.

Cohen, G.H., Coon, G.A. (1953). Theoretical Consideration of Retarded Control. *Transactions of the ASME*, 75, 827-834.

Cvejn, J. (2009). Sub-optimal PID controller settings for FOPDT systems with long dead time. *Journal of process control*, 19, 1486-1495.

Ho, W.K., Hang, C.C., Zhou, J.H. (1995). Performance and gain and phase margins of well-known PI tuning formulas. *IEEE Transactions on Control Systems Technology*, 3(2), 245-248.

Kiong, T.K., Quing-Guo, W., Chiech, H.Ch., Hägglund, T.J. (1999). *Advances in PID Control*. Springer Verlag.

Lopez, A.M., Murrill, P.W., Smith, C.L. (1967). Controller Tuning Relationships Based On Integral Performance Criteria. *Instrumentation Technology*, 14 (11).

Oldenbourg, R.C., Sartorius, H. (1956). A Uniform Approach to the Optimum Adjustments of Control Loops. In: *Frequency Response*, The Macmillan Co.

Rivera, D.E., Morari, M., Skogestad, S. (1986). Internal Model Control 4. PID Controller Design. *Industrial & Engineering Chemistry Process Design and Developement*, 1, 252-265.

Wang, F.S., Juang, W.S., Chan, C.T. (1995). Optimal tuning of PID controllers for single and cascade control loops. *Chemical Engineering Communications*, 132, 15-34.

Ziegler, J.G., Nichols, N.B. (1942). Optimum Settings for Automatic Controllers. *Transactions of the ASME*, 64, 759-768.

Haalman, A. (1965). Adjusting controllers for deadtime processes. *Control Engineering*, July-65 , 71-73.

O´Dwyer, A. (2003). *Handbook of PI and PID Tuning Rules*. Imperial College Press.