Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation, and Mathematics

PROCEEDINGS

of the 18$^{\mathrm{th}}$ International Conference on Process Control

Hotel Titris, Tatranská Lomnica, Slovakia, June 14 – 17, 2011

ISBN 978-80-227-3517-9

http://www.kirp.chtf.stuba.sk/pc11

Editors: M. Fikar and M. Kvasnica

Ogonowski, Z.: Two-State Bilinear Predictive Control for Hot-Water Storage Tank, Editors: Fikar, M., Kvasnica, M., In *Proceedings of the 18th International Conference on Process Control*, Tatranská Lomnica, Slovakia, 408–414, 2011.

Full paper online: http://www.kirp.chtf.stuba.sk/pc11/data/abstracts/060.html

Two-state bilinear predictive control for hot-water storage tank

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Abstract: The paper presents original predictive algorithm for use in two-state (or binary) input control of nonlinear systems which are described with state-constrained bilinear models. It is shown in the paper, that instead of non-linear continuous-time model, non-stationary linear discrete-time model can be used to predict the system response. On the other hand, state constrains can be attached to the criterion index to be minimized in the predictive control law. This inclusion assures the closed-loop stability of the control system and simplifies minimization problem. The proposed algorithm is particularly valuable for applications in heating systems where bilinearity follows from the heat exchange due to flow of liquid medium and constrains concern temperature regime. Application of the algorithm to control a hot water tank is presented in the paper. The tank is modeled with stratified model.

Keywords: bilinear models, predictive control, non-linear state observer, stratified models, hot-water tank.

1. INTRODUCTION

High hopes that were associated with non-linear predictive control to be a general control methodology proved to be futile at the turn of the century. Great ferment that the works of Michalska and Mayne (e.g. Michalska and Mayne (1993)) raised in the middle of nineties collapsed after confrontation with requirements of a real-world applications. The biggest problem posed optimization task which needs to be solved in every sampling period thus the applications have been restricted to slow processes Kwon and Han (2005). This problem was clearly stated at the end of nineties (Allgower and Zheng (2000), "Nonlinear Model Predictive Control: Challenges and Opportunities" by D. Mayne, pp. 23–44), and still remains unsolved. On the other hand special cases of nonlinearities has been studied meanwhile Rossiter (2004). One of the most deeply explored is the case of linear dynamics and input/state constrains Maciejowski (2002).

In the same spirit this paper explores bilinear systems with state constrains and two-state input signals as yet another special case of nonlinear system to be controlled. There are number of processes being modeled with bilinear models. The most important group of such systems form heat transfer processes where the energy is transported with liquid fluids ASHRAE (2009). The bilinear model of heat exchange arises due to states (temperatures) are multiplied by the control signal (liquid flow). Additionally, states are constrained by technological requirements and inputs are constrained because heat sources and pumps can be only switched on or off. Usually control systems apply simple relay controllers where so called cut-off method allows for compliance with constrains. Quality of the relay-based control can be hardly improved. There are only few knobs to be used as hysteresis or dynamical corrections Skoczowski (1981).

The paper presents original predictive algorithm for use in two-state input control of nonlinear systems which are described with state-constrained bilinear models.

Bilinear models are described in sec. 2. Based on these models the predictive control algorithm with two-state input and state constrains is derived in sec. 3. Discrete in time nature of predictive control needs discretization of the bilinear model. It is shown in sec. 3, that instead of non-linear continuous-time model, non-stationary linear discrete-time model is obtained. This technique is similar to so called successive linearization (e.g. in L. Magni and Allgower (2009), M. Cannon et.all. 'Successive Linearization NMPC for a Class of Stochastic Nonlinear Systems', pp. 249-262). However, proposed method uses exact model and does not impose linearization errors. State prediction on the assumed horizon is made on the basis of obtained model. It follows form the general theory of nonlinear predictive control Rossiter (2004) that closed-loop stability is assured by augmenting the criterion function to be minimized with final state weighting. This is done here by inclusion of the constrains into the criterion function as a penalty term.

Sec. 4 presents state observer for bilinear systems. This is the case of application example namely heating systems where bilinearity follows from the heat exchange due to flow of liquid medium and constrains concern temperature regime. Application of the algorithm to control a hot water tank is presented in sec. 5. The tank is modeled with so-called stratified model. Usually it is not possible to measure all states which follow from the stratification thus observer is necessary.

2. BILINEAR PROCESSES

Bilinear systems are the special nonlinear systems where linearity concerns separately state and control variables but not jointly. The general form of the bilinear model can be represented by the following:

$$\dot{X}(t) = AX(t) + B_0U(t) + \sum_{k=1}^{m} B_kX(t)u_k(t) + E(t) \quad (1)$$

where $X \in R^n$ and $U \in R^p$ (u_k represents k-th element of U). It is clear that $A \in R^{n\times n}$, $B_0 \in R^{n\times p}$ and $B_k \in R^{n\times n}$, $k = 1, 2, \ldots, m$. Operating point is assumed zero (the model (1) represents deviations from the operating point). Term $E(t)$ represents disturbances. It is assumed, that after sampled, disturbances can be modeled by white noise. This assumption allows for optimal in mean-square sense prediction of the state by using the model (1) with disturbances term omitted (optimal prediction of white noise is equal to zero).

Modern system theory made possible and stimulated expansion and deepening of research so that the intrinsic limits of linear models appear more and more evident. There are number of disciplines where bilinear models found applications e.g. industrial processes, biology, economics, ecology agriculture etc. This type of nonlinear dynamical models have been rigorously explored in the last three decades. It has been shown Mohler (1991) that bilinear systems are better controllable in general then linear systems. They offer better possibility in control performance. Still interest in these systems is very high. The structural theory is fairly well established and in particular there are several satisfactory contributions on controllability, mainly for homogeneous in the state bilinear systems. Also, with respect to mathematical modeling problems, the available results are quite definite.

3. PREDICTIVE TWO-STATE CONTROL OF BILINEAR SYSTEM WITH CONSTRAINS

In two-state control it is assumed that elements u_k of the control vector variable U can achieve only 0 or 1 value. Inequality state constrains are also involved, and can be expressed in general form as

$$\Omega X(t) \leq X_{con}. \quad (2)$$

Matrix Ω allows to easily limit on the maximum value of state variable (e.g. temperature cut-off) as well as the value of the acceptable range of states (e.g. output temperature of the heating system). Control predictive algorithm is formulated as discrete in time and zero-order holder of the control signal is assumed. The essence of the predictive control algorithm synthesis is solving of the optimization task in every sampling period. The objective function of the optimization task is formulated as a difference between predicted state trajectory and reference trajectory (usually equal to assumed set-points in the future) according to assumed prediction horizon. The optimizing criterion is the function of future controls, however, after optimization task is solved, only first element of the solution (nearest control) is applied and the whole procedure is repeated in the next sampling period (receding horizon technique).

Let the current moment in time is denoted by t_i, and sampling period T_s. Then: $t_{i+j} = t_i + j \cdot T_p$. Usually objective function is defined in the following quadratic form:

$$J(U(t_{i+j})|_{j=0,1,\ldots,N-1}) = \sum_{j=1}^{N} e_x^T(t_{i+j})Qe_x(t_{i+j}) + \\ + U^T(t_{i+j-1})RU(t_{i+j-1}) \quad (3)$$

where

$$e_x(t_{i+j}) = X(t_{i+j} - X_{sp}) \quad (4)$$

is j-step prediction of the difference between states X and their set-points X_{sp}. Positive (semi)definite matrices Q and R as well as the prediction horizon N form the algorithm's parameters. Constrains (2) of the optimization task should be fulfilled in every sampling period t_{i+j}, $j = 1, 2, \ldots, N$. Obviously, in general it is not possible to assure the existence of such control sequence $U(t_{i+j})|_{j=0,1,\ldots,N-1}$, that the constrains are fulfilled because initial conditions can be out of the constrains. Thus it is much simpler to include the constrains into the criterion function and allows penalty method for searching the optimal solution. This also simplifies the searching algorithm because the optimization task is now constrains-free. Finally, the criterion function takes the form

$$J(U(t_{i+j})|_{j=0,1,\ldots,N-1}) = \sum_{j=1}^{N} e_x^T(t_{i+j})Qe_x(t_{i+j}) + \\ + U^T(t_{i+j-1})RU(t_{i+j-1}) + \varphi(\Omega X(t_{i+j}) - X_{con}) \quad (5)$$

where φ is scalar penalizing function with the vector argument equal to exceeding the limits.

The above formulation of the predictive control algorithm allows for simple inclusion of requirements to keep the states within the proper range. There are two ways to do that:

- Determine set-points for the certain state and choose the proper weighting matrix Q in (5) depending on the role of the state in the system.
- Form the constrains (2) in such a way, that the range of certain state is properly narrowed.

The predicted states in objective function (5) should be determined from the model (1). Zero-order holder allows for the following representation of (1):

$$\dot{X}(t) = \left(A + \sum_{k=1}^{m} u_k(t_{i+j-1})B_k\right) X(t) + B_0U(t), \\ t \in [t_{i+j-1}, t_{i+j}]. \quad (6)$$

Equation (6) is linear and its solution at the end of the sampling period is as follows:

$$X(t_{i+j}) = \Phi_{i+j-1}X(t_{i+j-1}) + \Gamma_{i+j-1}U(t_{i+j-1}) \quad (7)$$

where

$$\Phi_{i+j-1} = e^{\left(A+\sum_{k=1}^{m} u_k(t_{i+j-1})B_k\right)T_p} \quad (8)$$

$$\Gamma_{i+j-1} = \int_0^{T_p} e^{\left(A+\sum_{k=1}^{m} u_k(t_{i+j-1})B_k\right)\tau} d\tau B_0. \quad (9)$$

Starting with initial state $X(t_i)$ the succeeding iterations are performed according to (7 – 9) for $j = 1, 2, \ldots, N$ to determine the whole trajectory of the state on the horizon N.

Number of possible control vector values on the horizon N is equal to $m \cdot 2^N$. If the sampling period is not to short then the prediction horizon need not to be large and the optimization task can be solved by bruteforce method. Similar approach was used in Ogonowski (2011b)

It should be emphasized that in every sampling period matrices Φ_{i+j-1} and Γ_{i+j-1} have to be determined. These matrices depend on input signal and change in every step. Thus the model (7) is nonstationary. Calculation of Φ_{i+j-1} and Γ_{i+j-1} needs application of special algorithms e.g. squaring and scaling Higham (2005). If sampling period is short and complex calculations are not possible then simplified model can be applied by using Euler method of integration:

$$X(t_{i+j}) = A'_{i+j-1} X(t_{i+j-1}) + B'_0 U(t_{i+j-1}) \tag{10}$$

where

$$A'_{i+j-1} = \left(A + \sum_{k=1}^{m} u_k(t_{i+j-1}) B_k \right) T_p \tag{11}$$

$$B'_0 = B_0 T_p. \tag{12}$$

4. STATE OBSERVER FOR BILINEAR SYSTEM

To calculate state prediction it is necessary to start iteration of the model (7) or (10) with current measurement of the state $X(t_i)$. Often the only part of X is measured. Then the state observer is necessary. The theory of bilinear model state observer is well established (e.g. Hara and Furuta (1976)). Assume that s elements of X vector is measured. The state vector can be ordered to keep them on the top, to simplify the notation:

$$Y(t) = (I_s \; 0)\, X(t) \tag{13}$$

where I_s is s-dimensional unity matrix. Y represents then vector of measured states. Equation (1) can be factorized as follows

$$\dot{X}(t) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} X(t) + \begin{bmatrix} B_{0,1} \\ B_{0,2} \end{bmatrix} U(t) + \\ + \sum_{k=1}^{m} \begin{bmatrix} B_{k,11} & B_{k,12} \\ B_{k,21} & B_{k,22} \end{bmatrix} X(t) u_k(t) \tag{14}$$

where $A_{11}, B_{k,11} \in R^{s\times s}$, $A_{12}, B_{k,12} \in R^{s\times(n-s)}$, $A_{21}, B_{k,21} \in R^{(n-s)\times s}$, $A_{22}, B_{k,22} \in R^{(n-s)\times(n-s)}$, $B_{0,1} \in R^{s\times m}$, $B_{0,2} \in R^{(n-s)\times m}$. In Hara and Furuta (1976) it was proven, that if the following two conditions are kept

$$\mathrm{Re}\left[\mathrm{eig}\left(A_{22} + HA_{12}\right)\right] < 0 \tag{15}$$

$$B_{k,22} + HB_{k,12} = 0 \quad k = 1, 2, \ldots, m \tag{16}$$

then there exists state observer of minimal order which is realized with the following dynamical system:

$$\dot{Z}(t) = \hat{A}Z(t) + \hat{B}_0 Y(t) + \sum_{k=1}^{m} \hat{B}_k Y(t) u_k(t) + \hat{G}U(t) \tag{17}$$

$$\hat{X}(t) = \hat{C}Z(t) + \hat{D}Y(t) \tag{18}$$

where

$$\hat{A} = A_{22} + HA_{12} \tag{19}$$

$$\hat{B}_0 = A_{21} + HA_{11} - (A_{22} + HA_{12})H \tag{20}$$

$$\hat{B}_k = B_{k,21} + HB_{k,11} \tag{21}$$

$$\hat{G} = B_{0,2} + HB_{0,1} \tag{22}$$

$$\hat{C} = \begin{bmatrix} 0 \\ I_{n-s} \end{bmatrix} \tag{23}$$

$$\hat{D} = \begin{bmatrix} I_s \\ -H \end{bmatrix}. \tag{24}$$

It was proven that the error $\hat{X}(t) - X(t)$ and all its derivatives tends to zero independently on U and initial conditions $X(t_0)$ and $Z(t_0)$.

5. APPLICATION

The proposed algorithm is particularly valuable for applications in heating systems where bilinearity follows from the heat exchange due to flow of liquid medium. Constrains concern temperature requirements. Application of the presented predictive algorithm will be now shown on the example of 300 liters hot water tank. The tank is equipped with one heating coil pipe placed in the upper part of the tank. The coil is fed with on-off controlled boiler (16200 W) throughout water as a heating medium. The tank has been equipped with measurement system Ogonowski (2010) containing termo-elements and hot water flow meter. The tank is modeled with stratified model.

5.1 Model of the hot water tank

Hot water tank is a vertically standing cylinder equipped with M heating pipe coils distributed in different parts along the vertical axis. Cold water water enters the tank bottom and is charged on top. Thus temperature gradient occurs. After division of the cylinder onto S layers the basic heat balance can be written as follows:

$$Q_{wn} = \sum_{m=1}^{M} Q_{m(n)} - Q_{un} - Q_{sn}, \tag{25}$$

where Q_{wn} is the heat accumulated in the n-th layer, $Q_{m(n)}$ is the heat transmitted by the m-th source to the n-th layer, Q_{un} is the heat applied from n-th layer and Q_{sn} is the heat loses of the n-th layer to the surroundings. Let consider single layer which is driven with a heat source transmitting Q_p through the heating medium of the flow F_z with enter temperature T_{zi} and exit temperature T_{zo}. Could water has got the temperature on the input equal to T_{wi} and on the output T_{wo}. The tank is surrounded by the environment of the temperature T_{sur}. Heat exchange describes the following two differential state equations:

$$\rho C_w V \frac{dT_{wo}}{dt} = \rho C_w F_z (T_{zi} - T_{zo}) - \rho C_w F_w (T_{wo} - T_{wi}) - \\ - \frac{\lambda A}{d}(T_{wo} - T_{sur}), \tag{26}$$

$$\rho C_w V_w \frac{dT_{zi}}{dt} = Q_p - \rho C_w F_z (T_{zi} - T_{zo}) - \frac{\lambda_w A_w}{d_w}(T_{zi} - T_{sur}),$$

where ρ, C_w represents density and specific heat of the water respectively, V, V_w are the volumes of the layer and pipe coil, λ, λ_w are heat permeability coefficient of the tank wall and pipe coil respectively. In general, by division of the both sizes of (26) by $\rho C_w V$ and $\rho C_w V_w$, and taking into account heat exchange between layers, one can derive the following:

$$\frac{dT_{wo}^n}{dt} = b_1^n F_z^{n,m}(T_{zi}^n - T_{wo}^n) - b_2^n F_w(T_{wo}^n - T_{wo}^{n-1}) -$$
$$b_3^n(T_{wo}^n - T_{sur}) - b_4^n(T_{wo}^n - T_{wo}^{n-1}) + b_5^n(T_{wo}^{n+1} - T_{wo}^n), \quad (27)$$
$$\frac{dT_{zi}^m}{dt} = p_1^n Q_g^m - p_2^n F_z^m(T_{zi}^m - T_{wo}^n) - p_3^n(T_{zi}^m - T_{sur}),$$

where superscript n denotes number of layer, $n+1$ is the number of upper layer and $n-1$ is the number of lower layer. m is the number of heat source which is directly coupled with the n-th layer. In equation (27) physical coefficients has been exchanged with constants b and p. First evaluation of b and p can follow from physical meaning. The final ones, however, have to be identified because stratified model is simplification of the real plant which has got a distributed parameter nature. Additionally, dependent variable T_{zo} has been excluded from the above equations which is possible under assumption that heat transfer driving force is the average temperature T_{zi} and T_{zo} Marlin (1995).

5.2 Parameter identification

Simple method for identification of (27) model bases on distinguishing the periods of time where some parts of the model remain zero. This follows from specific of the model (27) e.g. if heat source or pump does not work then respective signals F_z or F_w are zero. The model (27) can be identified part by part with properly chosen data.

It is assumed three-layered structure of the model thus the model takes form:

$$\frac{dT_{wo}^3}{dt} = b_1^3 F_z(T_{zi} - T_{wo}^3) - b_2^3 F_w(T_{wo}^3 - T_{wo}^2) -$$
$$-b_3^3(T_{wo}^3 - T_{sur}) - b_4^3(T_{wo}^3 - T_{wo}^2), \quad (28)$$
$$\frac{dT_{wo}^2}{dt} = b_1^2 F_z(T_{zi} - T_{wo}^2) - b_2^2 F_w(T_{wo}^2 - T_{wo}^1) -$$
$$-b_3^2(T_{wo}^2 - T_{sur}) - b_4^2(T_{wo}^2 - T_{wo}^1) + b_5^2(T_{wo}^3 - T_{wo}^2), \quad (29)$$
$$\frac{dT_{wo}^1}{dt} = -b_2^1 F_w(T_{wo}^1 - T_{wi}) - b_3^1(T_{wo}^1 - T_{sur}) +$$
$$+b_5^1(T_{wo}^2 - T_{wo}^1), \quad (30)$$
$$\frac{dT_{zi}}{dt} = p_1 Q_g - p_2 F_z(T_{zi} - T_{wo}^3) - p_3(T_{zi} - T_{sur}). \quad (31)$$

Formulation (28)-(31) can be transformed to (1) with the following:

$$X = \begin{bmatrix} T_{wo}^1 \\ T_{wo}^2 \\ T_{wo}^3 \\ T_{zi} \end{bmatrix}, \quad U = \begin{bmatrix} F_z \\ Q_g \end{bmatrix},$$

$$A = \begin{bmatrix} -(b_3^1 + b_5^1) & b_5^1 & 0 & 0 \\ b_4^2 & -(b_3^2 + b_4^2 + b_5^2) & b_5^2 & 0 \\ 0 & b_4^3 & -(b_3^3 + b_4^3) & 0 \\ 0 & 0 & 0 & -p_3 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & p_1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -b_1^2 & 0 & b_1^2 \\ 0 & 0 & -b_1^3 & b_1^3 \\ 0 & 0 & p_2 & -p_2 \end{bmatrix}, \quad B_2 = 0_{4\times 4} \quad (32)$$

$$E(t) = \begin{bmatrix} b_3^1 T_{sur} + b_2^1 F_w(T_{wi} - T_{wo}^1) \\ b_3^2 T_{sur} + b_2^2 F_w(T_{wo}^1 - T_{wo}^2) \\ b_3^3 T_{sur} + b_2^3 F_w(T_{wo}^2 - T_{wo}^3) \\ p_3 T_{sur} \end{bmatrix}.$$

Note, that disturbance vector $E(t)$ is added to the right hand side of (1). E contains two components. The first depends on T_{sur} and changes sufficiently slow to be accepted as constant. The second depends on F_w and changes much faster (see Figure 1). Analysis of F_w shows its white character. This justifies assumption, that disturbances term can be omitted in prediction.

It can be easily verified that if the heating is off ($Q_p, F_z = 0$) and no hot water is use ($F_w = 0$) then all state are equal to T_{sur}. Assuming $T_{sur} = const$ one can use deviation model where X means deviations from T_{sur}.

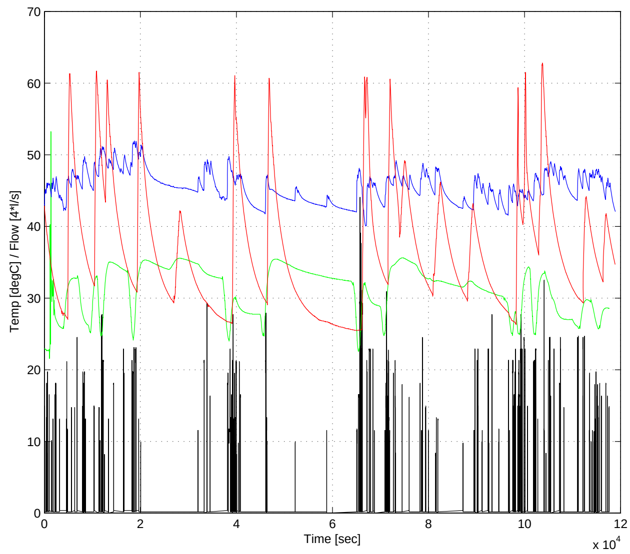

Fig. 1. Example of data for identification. T_{wo}^3 - blue, T_{wo}^2 - green, T_{zi} - red, F_w - black.

Figure 1 presents example of data. There are three temperatures measured: hot water (output of the tank) T_{wo}^3 – blue line, temperature at half height of the tank T_{wo}^2 (green) and temperature of the heating medium T_{zi} (red). Lower part of the tank keeps constant temperature $T_{wo}^1 = 10^o$C which need not to be measured. $T_{sur} = 25.5^o$C was assumed constant as well. On-off control signal were boiler power $Q_g = 0$ or 16200 and heating medium flow $F_z = 0$ or 0.5. It is interesting to notice the behavior of T_{wo}^3 temperature: if the heating is off (T_{zi} decreases) and hot water use appears (black line) then T_{wo}^3 increases for some time while T_{w0}^2 decreases. This phenomenon follows form placement of the measurement element – close, but outside of the tank. If the pump is off the pipe gets colder despite of the high inner temperature. After disturbance occurs pipe gets warmer despite temperature of cold water at the input of the tank is much lower then the inner temperature.

After carefully chosen periods it became possible to determine parameters of three-layer model Ogonowski (2011a) as presented in Table 1.

Table 1. Parameters of three-layer model.

Layer/ Parameter	$n=1$	$n=2$	$n=3$
b_1	0	0.019	0.025
b_2	0.73	0.071	0.067
b_3	0.00005	0.00093	0.0058
b_4	0	0.00076	0.0049
b_5	0.00001	0	0
p_1	0.13		
p_2	0.015		
p_3	0.005		

5.3 Standard rely control

In practice, standard control system of hot water tank uses two relays. The first (with hysteresis) stabilizes T_{wo}^2 on the prespecified set-point $T_{wo,sp}^2$. The second realizes so called cut-off algorithm: if T_{zi} exceeds $T_{zi,cut}$ then the boiler is switched off, however pump is still on until the first rely is on. This very simple algorithm is robust and ensures the maintenance of hot water volume on some level due to middle temperature is stabilized instead of the output one. The only drawback seems indirect stabilization of the output temperature. Thus, $T_{wo,sp}^2$ has to be properly chosen (usually by trial and error method).

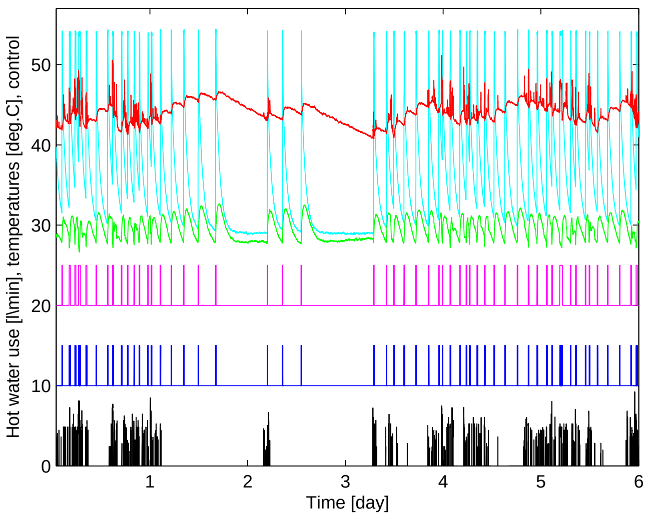

Fig. 2. Results of the standard on-off control performance. T_{wo}^3 - red, T_{wo}^2 - green, T_{zi} - cyjan, F_w - black, Q_p - blue (scaled to 10-15), F_z - magenta (scaled to 20-25).

Figure 2 demonstrates the results of the standard control under real-world operations. This means not only real-world experimentation in the environment sense, but also that the control system was tested during normal using of the tank. Disturbances (hot water use) caused a decrease in output temperature and control system reaction. Set point for middle temperature is $T_{wo,sp}^2 = 28^o$C and is kept properly (green). Mean value of the output temperature T_{wo}^3 is equal to 46^{o}C (red). Variance if T_{wo}^3 is relatively large, however, one should remember that T_{wo}^3 is controlled indirectly.

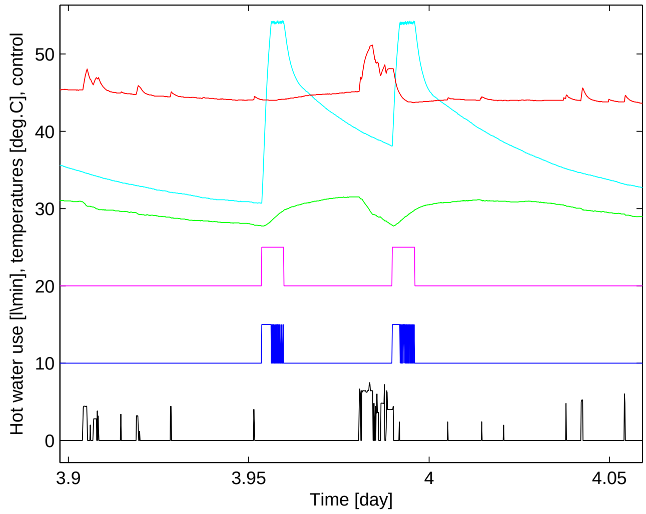

Fig. 3. Detail of the figure 2.

Detail of Figure 2 is presented in Figure 3 and explains on-off algorithm performance. After T_{wo}^2 reached 28^{o}C (green), pomp F_z (magenta) and boiler Q_p (blue) gets on (time 3.98). T_{zi} (cyjan) increases fast and after reached cut-off temperature $T_{zi,cut} = 54^o$C signal Q_p starts switching. In this time F_z remains on because $T_{wo}^2 < T_{wo,sp}^2 = 28^o$C. After T_{wo}^2 reached 30^{0}C (28+2^{o}C of hysteresis) both control signals are off. Then, due to succeeding use of hot water (black disturbances) T_{wo}^2 decreases and the next reaction of the controller takes place (time about 4.03). Note the phenomenon of temporary increase of T_{wo}^3 after hot water use.

5.4 Predictive control

Observer. Standard relay control does not need T_{wo}^3 measurement (only T_{wo}^2 and T_{zi} are necessary). Thus the tanks are not equipped with inner (i.e. placed in the probe) measuring thermoelement. Even if the outer measurement is possible (i.e. using clip-on temperature sensor) the phenomenon described above disturbs the result significantly thus the measurement can be hardly used for control. To conclude, observer of T_{wo}^3 is necessary. In fact, there is also T_{wo}^1 to be observed because it is not measured. However, bottom part of the tank has constant temperature, or it changes in significantly small range, thus the result of the observation has little influence on the control system.

According to (13) the states are rearranged to the following form:

$$X = \begin{bmatrix} T_{wo}^2 \\ T_{zi} \\ T_{wo}^3 \\ T_{wo}^1 \end{bmatrix} \tag{33}$$

and vector of measurement Y can be written as:

$$Y(t) = (I_2 \ 0)X(t) \tag{34}$$

where I_2 is unity matrix of 2×2 size. The task of the observer is to determine

$$\hat{X} = \begin{bmatrix} T^2_{wo} \\ T_{zi} \\ \hat{T}^3_{wo} \\ \hat{T}^1_{wo} \end{bmatrix} \tag{35}$$

which elements $\hat{T}^3_{wo}$ and $\hat{T}^1_{wo}$ tends sufficiently fast to T^3_{wo} and T^1_{wo} independently on initial conditions and disturbances. The matrices of the model (14) are as follows

$$A_{11} = \begin{bmatrix} -(b_3^2 + b_4^2 + b_5^2) & 0 \\ 0 & -p_3 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} b_5^2 & b_4^2 \\ 0 & 0 \end{bmatrix}$$
$$A_{21} = \begin{bmatrix} b_4^3 & 0 \\ b_5^1 & 0 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -(b_3^3 + b_4^3) & 0 \\ 0 & -(b_3^1 + b_5^1) \end{bmatrix} \tag{36}$$

$$B_{0,1} = \begin{bmatrix} 0 & 0 \\ 0 & p_1 \end{bmatrix}, \quad B_{0,2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{37}$$

$$B_{1,11} = \begin{bmatrix} -b_1^2 & b_1^2 \\ 0 & -p_2 \end{bmatrix}, \quad B_{1,12} = \begin{bmatrix} 0 & 0 \\ p_2 & 0 \end{bmatrix}$$
$$B_{1,21} = \begin{bmatrix} 0 & b_1^3 \\ 0 & 0 \end{bmatrix}, \quad B_{1,22} = \begin{bmatrix} -b_1^3 & 0 \\ 0 & 0 \end{bmatrix} \tag{38}$$

and $B_{2,11} = B_{2,12} = B_{2,21} = B_{2,22} = 0_2$, where 0_2 is zero matrix of 2×2 size.

It can be easily checked that the condition (16) takes the form:

$$H = \begin{bmatrix} h_{11} & \dfrac{b_1^3}{p_2} \\ h_{21} & 0 \end{bmatrix} \tag{39}$$

Characteristic equation of a matrix

$$\hat{A} = A_{22} + HA_{12} = \begin{bmatrix} h_{11}b_5^2 - b_3^3 - b_4^3 & h_{11}b_4^2 \\ h_{21}b_5^2 & h_{21}b_4^2 - b_3^1 + b_5^1 \end{bmatrix} \tag{40}$$

has got the following form

$$\lambda^2 + \alpha\lambda + \beta = 0 \tag{41}$$

where:

$$\alpha = -h_{11}b_5^2 + (b_3^3 + b_4^3) - h_{21}b_4^2 + (b_3^1 + b_4^3), \tag{42}$$

$$\beta = h_{11}b_5^2(b_3^1 + b_5^1) + h_{21}b_4^2(b_3^3 + b_4^3) + (b_3^1 + b_5^1)(b_3^3 + b_4^3) \tag{43}$$

It follows from the Hurwitz criterion that the condition (15) is fulfilled if

$$\alpha > 0 \quad \text{i} \quad \beta > 0 \tag{44}$$

This can be transformed into two cases:

$$\text{If} \quad b_5^2 = 0 \quad \begin{cases} h_{11} & -\text{arbitral} \\ h_{21} < \dfrac{b_3^1 + b_5^1}{b_4^1} \end{cases} \tag{45}$$

$$\text{If} \quad b_5^2 \neq 0 \quad \begin{cases} h_{11} < -\dfrac{b_4^2}{b_5^2}h_{21} + \dfrac{b_3^3 + b_4^3 + b_3^1 + b_5^1}{b_5^2} \\ h_{11} < -\dfrac{b_4^2(b_3^3 + b_4^3)}{b_5^2(b_3^1 + b_5^1)}h_{21} + \dfrac{b_3^3 + b_4^3}{b_5^2} \end{cases} \tag{46}$$

Derivation of (45) and (46) used fact, that $b_i^k \geq 0$. Using parameters of the model given in Table 1 one obtains: $h_{21} < 0.0789$ and h_{11} to be freely chosen. The choice influences convergence of the observer. Speed of the convergence follows from eigenvalues of $\hat{A}$. In the case discussed ($b_5^2 = 0$ – see Table 1), the eigenvalues are equal to:

$$\begin{aligned} \lambda_1 &= -b_3^3 - b_4^3 \\ \lambda_2 &= h_{21} - b_3^1 - b_5^1 \end{aligned} \tag{47}$$

It is clear form (47) that λ_1 does not depend on H, thus the speed of convergence can be shaped to a small extent by changing only λ_2. Finally the following values has been chosen:

$$H = \begin{bmatrix} 0 & 1.667 \\ -10 & 0 \end{bmatrix}. \tag{48}$$

Example of application. Set-points have been determined for the states as follows:

$$X_{sp} = \begin{bmatrix} T^2_{wo,sp} \\ T_{zi,sp} \\ T^3_{wo,sp} \\ T^1_{wo,sp} \end{bmatrix} = \begin{bmatrix} 28 \\ 54 \\ 46 \\ 10 \end{bmatrix}. \tag{49}$$

Sampling time has been chosen $T_s = 1$ min. The model (1) has been discretized with simple Euler method (10). Constrains can be summarized as

$$\Omega\hat{X} = [0 \ \ 1 \ \ 0 \ \ 0] \begin{bmatrix} T^2_{wo} \\ T_{zi} \\ \hat{T}^3_{wo} \\ \hat{T}^1_{wo} \end{bmatrix} \leq 54^o\text{C} \tag{50}$$

One can notice that the control system influences of T_{zi} in two ways (set-point and constrains). This problem does not disappear after inclusion of the constrains into the criterion function (5). However, proper choice of the weighting matrix Q transfres the responsibility of T_{wzi} control on the penalizing function:

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad q_1 \geq 0, \quad q_2 \geq 0. \tag{51}$$

Note, that the second row and column is zero. Control weighting matrix is assumed to be diagonal as well

$$R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}, \quad r_1 \geq 0, \quad r_2 \geq 0. \tag{52}$$

Penalizing function is assumed to the Heaviside one

$$\varphi\left(\Omega\hat{X}(\cdot) - X_{con}\right) = \alpha 1(T_{zi}(\cdot) - 54^o\text{C}), \quad \alpha > 0. \tag{53}$$

The above formulated algorithm has bee tuned by trial and error method using simulations which has been conducted with the disturbances that had been measured during relay control experiment (see Figure 2). The results of tuning are as follows: $N = 4$, $\alpha = 150$, $q_1 = q_2 = 1.36$, $r_1 = r_2 = 0.12$. Figure 4 presents the results of the predictive control algorithm performance.

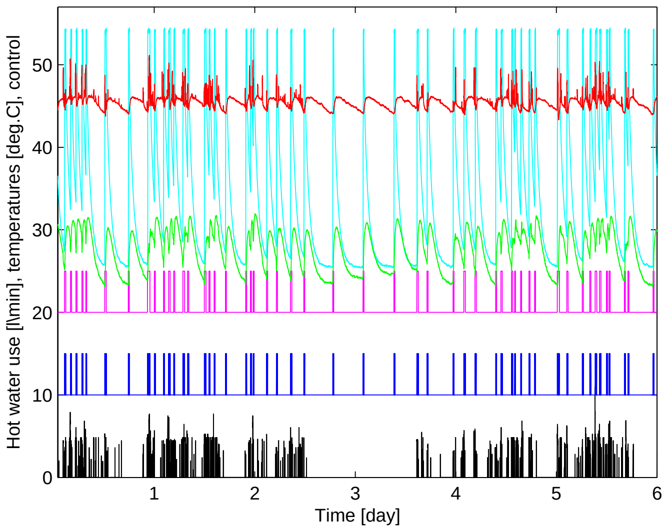

Fig. 4. Results of the predictive control algorithm performance. T^3_{wo} - red, T^2_{wo} - green, T_{zi} - cyjan, F_w - black, Q_p - blue (scaled to 10-15), F_z - magenta (scaled to 20-25).

Obviously, there is no possibility to use the same disturbance signal as in relay case, because experiment in real-life environment can not be repeated. However, it can be noted significantly better stabilization of the output T^3_{wo} temperature of the hot water (red line). On the other hand, stabilization of the middle T^2_{wo} temperature is worse (green) but yet this state is not important from the user needs view point. This can be seen e.g. between 2.5 and 3.5 [day]. Even in the absence of hot water outlets, reaction of the control algorithm takes place. This is due to the existence of feedback from observed T^3_{wo} which decreases because the tank cools down.

One would expect increase of the fuel consumption due to more frequent reaction of the control system when compare with the standard relay controller. This is not true. After much longer tests it became clear that predictive control is significantly economical. Long term observations proved about 9.5% fuel save when compare with relay control. The reason probably follows from the fact that predictive control takes into account energy price while minimizing objective function due to the term $U^T RU$. Standard controller does not take into account energy consumption at all.

6. CONCLUSION

Predictive control algorithm with state constrains allows for much better control performance then standard relay controllers. However, it is paid for with difficulty of tuning. There are number of parameters that should be properly chosen. What is more, quality of control depends on the quality of the model, because the prediction depends directly on model accuracy and indirectly on precision of observer which in turns depends on the model. Two further directions of research seems necessary to be undertaken. The first is multilayering of the control system structure which allows for application of upper-layer optimization of the controller parameters and operating point. The optimization can directly take into account fuel consumption. The second direction is adaptation of the model (or model self-tuning). It is possible due to structure of the model is known (stratification). Additional problem could be robustness of the control system on the model inaccuracy.

The paper presents application of the proposed algorithm to three layered model. The volume of the tank being tested allows for such stratification. Large industrial tanks need more precise stratified model which are build with greater number of layers. The theory, however remains the same and can be directly used.

ACKNOWLEDGMENTS

The work has been supported by the grant of Institute of Automatic Control, Silesian University of Technology, Gliwice, Poland. This support is very gratefully acknowledged.

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