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Bode Plots in Maxima Computer Algebra System

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Abstract: The aim of the paper is to demonstrate possibilities of open software environment Maxima in educational process at technical universities whereby our attention is dedicated to the teaching of Bode plots. The developed procedure for drawing its asymptotic approximation can be used both for checking results on the base of the entered system transfer function and also for self testing purposes. In addition, the results were used for the building of web application that will be used in frame of the subject Control Theory.

Keywords: computer algebra system, open source, Maxima, Bode plots, control theory

1. INTRODUCTION

Computer algebra systems are systems that enable to solve mathematical tasks either in numerical or symbolical way. Following Internet resources one can find several products that can be divided into two main groups: proprietary software that is a product of some company taking care about the whole development and support and open software developed usually by an enthusiastic person or group of persons that is available for free.

We decided to devote our attention to open alternative since it helps to solve problems connected with legal purchase of software that is used. In the case of proprietary products students usually use another version of software at the faculty and another version (very often illegal) at home. After finishing their study the proprietary software that is devoted to specific purposes like mathematical calculations are is too expensive for paying licences and keeping it regularly updated. Therefore they stop to use it. Open software can overcome these problems and therefore it could be good if students would know about its existence already during their stay at university.

Looking for the best alternative of computer algebra system (CAS) we had several requirements. The selected software should be open and enable at least

- numerical and symbolical calculations,
- simplification and processing of algebraic expressions,
- matrix computations,
- solution of a system of linear, nonlinear and differential equations,
- differentiate and integrate functions,
- graphical presentations of results,
- simple programming.

Except of that the selected CAS should have good ongoing support, cooperative community, continuing development and it would be welcomed if it could be supported both for Windows and Unix/Linux operating system. After considering several software environments (Axiom, Maxima, SymPy, Sage, Yacas, etc.) we decided to choose Maxima computer algebra system that can be an open alternative for such proprietary programs as Maple, Mathematica or Matlab with Symbolic Toolbox are.

2. BODE PLOTS

A Bode plot is a plot showing one of graphical representation of frequency response characteristics for linear time invariant system usually described by a transfer function. The sinusoidal transfer function of any linear system (obtained by substituting $j\omega$ for s in the transfer function of the system) is characterized by its magnitude and phase angle with frequency as parameter. Therefore the Bode plots are represented by two separate plots - one expressing the magnitude of the frequency response gain (magnitude vs. frequency plot) and another one expressing the frequency response phase shift (phase vs. frequency). Standard Bode plots are logarithmic on the frequency axis, and plot the magnitude in dB's (decibels) and phase in degrees.

Using computers drawing of plots can be done by simple entering many values for the frequency, calculating the magnitude and phase at each frequency and displaying them.

However, there are reasons to develop a method for sketching Bode diagrams manually. By drawing the plots by hand you develop an understanding about how the location of poles and zeros influences the shape of the plots. With this knowledge you can predict how a system behaves in the frequency domain by simply examining its transfer function. On the other hand, if you know the shape of transfer function that you want, you can use your knowledge of Bode diagrams to generate the transfer function.

Therefore our aim was not only to draw Bode plots of linear dynamical systems but to visualise also their asymptotic approximations. The main advantage of using Bode plots is that multiplication of magnitudes can be converted into addition.

To sketch asymptotic approximates of Bode diagram it is necessary to draw separate asymptotic curves for each of the transfer function factors

- gain,
- integral and derivative factors,
- first-order factors,
- quadratic factors.

The composite curve (both for magnitude and phase as well) is obtained by adding algebraically the individual curves. In order students could check their results, the developed Maxima procedure tries to offer results in the way as students obtain them manually. Actually, any change in the slope of the magnitude curve is made only at the break frequencies (break frequency or corner frequency is the frequency at which two asymptotes meet) of the transfer function. Therefore, instead of drawing individual magnitude curves and adding them up, it is possible to sketch the magnitude curve without sketching individual curves. In similar way, it is possible to proceed also in the case of the phase curve.

3. MAXIMA IMPLEMENTATION

Computation and visualisation of Bode diagram in Maxima can be realised very easily by using standard functions `bode_gain()` for magnitude plot and `bode_phase()` for phase plot. The input of both functions is a transfer function of a system. These functions were already created by other persons and there is no need to change them.

Our interest was first of all dedicated to the creation of Bode diagram asymptotic approximations. As it was already told the created procedure tries to demonstrate all steps of the manual graphical sketching that include

- transformation of the transfer function to the normalized form

$$F(s) = K\frac{s^p \prod(T_i s+1)\prod\left(\mathrm{T_k}^2 \mathrm{s}^2 + 2\mathrm{b_k}\mathrm{T_k}\mathrm{s}+1\right)}{s^r \prod\left(T_j s+1\right)\prod\left(\mathrm{T_l}^2\mathrm{s}^2+2\mathrm{b_l}\mathrm{T_l}\mathrm{s}+1\right)}$$

- separation of all transfer function factors (gain, integral and derivative factors, first-order factors, quadratic factors)
- calculation of break frequencies
- visualisation of asymptotic approximation of magnitude and phase plots for each separate factor
- adding up the previous results and drawing the final asymptotic approximation for both Bode plots.

Similar to standard Bode functions the created Maxima function has one input – the transfer function of the system. The function output generates asymptotic approximations of frequency responses together with all break frequencies.

4. WEB APPLICATION

It is to say that Maxima is the software that was developed only for the use on local computer. Since we want to use it as an engine for driving our web application it was necessary to prepare tools that would enable to communicate with Maxima via Internet. This functionality was already realised and was described in our previous papers (see e.g. Magyar et all, 2009). The developed implementation enables to exchange commands and data between Maxima software installed on linux server and our web application only on the base of created php functions.

The front end of the application was created using standard technologies as HTML, CSS and JavaScript. For the data exchange the JSON format was used. For plotting of the graphical dependencies we used free Javascript plotting library for jQuery that is called jqPlot. It produces graphical plots of arbitrary datasets on the client-side.

4.1 Basic functionality

The developed web application enables to draw asymptotic approximations of Bode plots. During university courses students learn to sketch these approximations in order they would be able to estimate behaviour of dynamical systems also without using computer.

The considered system is defined by the transfer function that is entered using coefficients of the transfer function numerator and denominator. In Fig. 1 the system with the transfer function

$$F(s) = \frac{\mathrm{s}+1}{\mathrm{s}^2+5\mathrm{s}+6} = \frac{\mathrm{s}+1}{(\mathrm{s}+2)(\mathrm{s}+3)} = \frac{1}{6}\frac{\mathrm{s}+1}{\left(\frac{\mathrm{s}}{2}+1\right)\left(\frac{\mathrm{s}}{3}+1\right)}$$

is started to be solved.

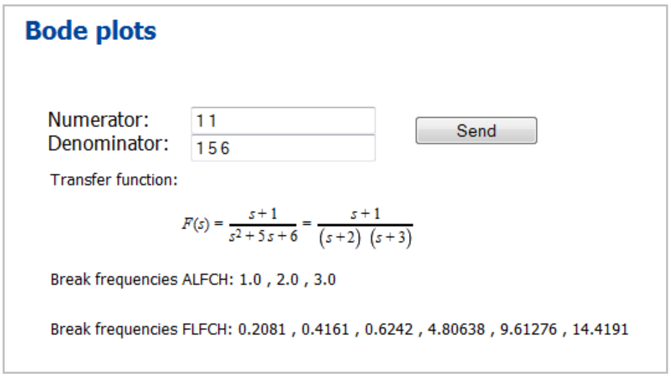

Fig. 1. Form for entering the input coefficients

After submitting the entered parameters, the user input is sent for processing to Maxima where break frequencies and corresponding slopes of asymptotes are calculated. The graphical result is displayed again in web browser (Fig.2). The advantage is that user can display resulting frequency response, final asymptotic approximation of Bode plots or only asymptotic approximation of chosen separate factor. In this way student can check all steps of own manual procedure.

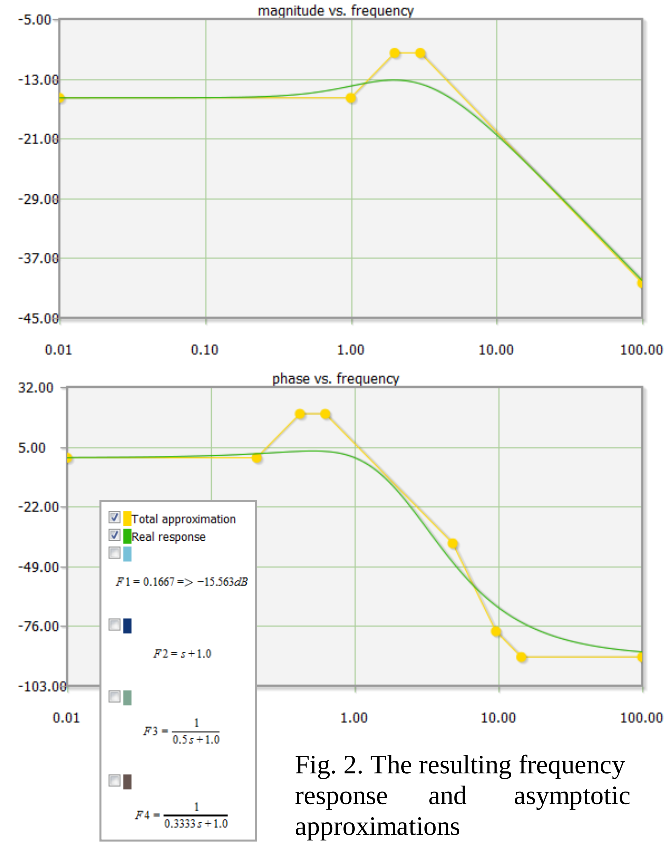

Fig. 2. The resulting frequency response and asymptotic approximations

4.2 Self testing

The developed application can also be used in other way. Students should be able not only to sketch Bode plots but also to analyse the model of the system according to sketched frequency characteristics. Therefore we prepared the modification of the application that can be used for self testing of students. Instead of entering parameters of the transfer function by student the parameters are determined randomly by computer. Then, the resulting asymptotic approximation of the magnitude Bode plot is visualised to the student (Fig.3). According to break frequencies student can determine time constants of the system and according to the vertical position of the characteristics the gain of the system can be found. In this way student can determine the resulting transfer function that is after entering to the web page compared with the originally generated system description (Fig.4). Maxima computer algebra system is able to compare both transfer functions without any problem.

In the case of self testing we decided to concentrate only on transfer functions with real poles and real zeros. The transfer functions containing complex roots are not considered because of the problematic determination of a system damping from the graph. From similar reason we do not consider systems with non minimal phase. The transfer function that is taken into account has the following form

$$F(s) = K\frac{\prod_{i=1}^{m}(T_i s+1)}{s^r\prod_{j=1}^{n}(T_j s+1)}$$

whereby the introduced parameters can achieve the following values: $K = 1,\ 100$; $T_i, T_j = 0,\ 0.125,\ 0.2,\ 0.25,\ 0.5,\ 1$; $r = 0, 1, 2$; $m, n = 1, 2, 3$.

The parameters are restricted to the mentioned values because of their easier identification from the graphical presentation of the magnitude plot. We decided to use only values that can be read from the picture exactly and without any doubt. It is very convenient and useful if no values rounding has to be used. In addition, such restriction of values doesn't mean any limitation to the complexity of the task. It is more important to consider various structures of transient functions.

Following the considered values of parameters the considered transient function can achieve several forms e.g.

$$F_1(s) = \frac{K}{s(Ts+1)}$$

$$F_2(s) = \frac{K}{(Ts+1)^2}$$

$$F_3(s) = \frac{K}{(T_1 s+1)(T_2 s+1)(T_3 s+1)}$$

$$F_4(s) = \frac{K(T_1 s+1)}{s^2(T_2 s+1)}$$

Of course, we didn't introduce all possibilities. The number of transient functions should be sufficient for comprehension of the problem by student.

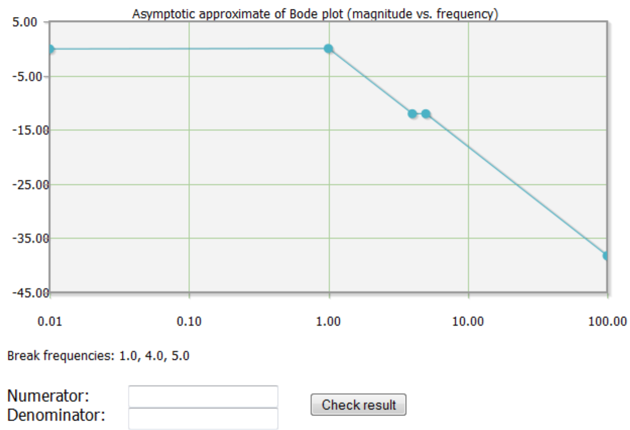

Fig. 3. Randomly generated asymptotic approximation of the magnitude Bode plot

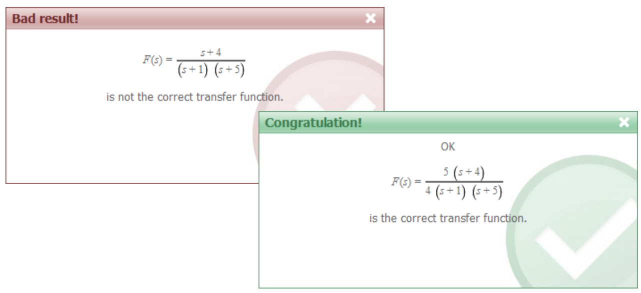

Fig.4. Verification of the result

5. CONCLUSIONS

In the paper we exploited a possibility to use Maxima software for solution of one problem from Control Theory.

The advantage is that the tool is available online via Internet for all interested users in any time they need. Since the presented topic is part of the study at probably all technical universities it presents the useful instrument in an educational process mainly in online and distance forms of education.

The last thing to mention is the fact that each version of the introduced application was prepared by regular students in frame of their individual projects. They like programming and together with it they learn control theory, too. The idea of "learning by doing" is in this case very actual and the results showed that it is good to continue in this direction also in future.

The application is a part of study materials that are available on the address:

http://obelix.urpi.fei.stuba.sk/~tar/pedagogika/tar1/

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REFERENCES

Cheever, E. (2005). What Bode Plots Represent, *url*: http://jegyzet.sth.sze.hu/ftp/!BSc/Szabalyozastechnika/BodePl.pdf

Jakab, F., Andoga, V., Kapova, L., Nagy, M. (2006). Virtual Laboratory: Component Based Architecture Implementation Experience, *Electronic computer and informatics*, September, Košice-Herľany, Slovakia.

Leão, C. P., A. E. Rodrigues (2004). Transient and steady-state models for simulated moving bed processes: numerical solutions. *Computers & Chemical Engineering* 28(9): 1725-1741.

Lutus P. (2007), Symbolic Mathematics Using Maxima, *url*: http://arachnoid.com/maxima/

Magyar, Z., T. Starý, L. Szolik, Ľ. Vörös, K. Žáková (2009). Modeling of Linear Dynamical Systems Using Open Tools. 10*th International Conference Virtual University*, December, Bratislava, Slovak Republic.

Ogata K. (1997). *Modern Control Engineering*, 3rd Edition, Prentice Hall London.

Resig J. and the jQuery Team (2009). jQuery Documentation. *url*: http://docs.jquery.com

Restivo, M. T., J. Mendes, A.M. Lopes, C.M. Silva, F. Chouzal (2009). A Remote Lab in Engineering Measurement, *IEEE Trans. on Industrial Electronics*, vol. 56, no.12, pp. 4436-4843.

Schauer, F., M. Ožvoldová, F. Lustig (2008). Real Remote Physics Experiments across Internet – Inherent Part of Integrated E-Learning. *Int. Journal of Online Engineering* (*iJOE*), 4, No 2.

Schmid, Chr. (2003). Internet - basiertes Lernen. *Automatisierungstechnik*, 51, No. 11, p. 485-493.

Turgeon, A. J. (2002) Implications of Web-Based Technology for Engaging Students in a Learning Society", *Journal of Public Service and Outreach* 2(2): 32-37.

Zolotová, I., M. Bakoš, L. Landryová (2007). Possibilities of communication in information and control systems. *Annals of the university Craiova, Series: Automation, Computers, Electronic and Mechatronic*, Vol.4(31), No.2, pp.163-168, ISSN 1841-062.

Žáková, K., M. Janotík (2004). Mathematical Modeling of Dynamic Systems: an interactive online lesson. 5*th int. conf.* "*Virtual University*", Bratislava, Slovakia.

Žáková, K. (2005). Control Theory - an interactive online course. 6*th int. conf.* "*Virtual University*", Bratislava, Slovakia.

Žáková, K., M. Sedlák (2006). Remote Control of Experiments via Matlab, *Int. Journal of Online Engineering* (*iJOE*), 2, No. 3.

jsMath: A Method of Including Mathematics in Web Pages, http://www.math.union.edu/~dpvc/jsMath/, Last modified: 10 Mar 2009.