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## Control of a Tubular Heat Exchanger

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**Abstract:** Using of a modified Smith predictor for compensation of measurable disturbances affecting a time-delay system is studied in this paper. The controlled system is a tubular heat exchanger, in which the kerosene is heated by hot water. The heat exchanger is a nonlinear system with time delay. The Smith predictor and the modified Smith predictor are used for control of the heat exchanger without and with disturbances. Obtained simulation results confirm that the modified Smith predictor with feed-forward compensation of measurable disturbances can improve the closed-loop control responses of the time delay systems with disturbances.

**Keywords:** time delay, disturbance, Smith predictor, modified Smith predictor, tubular heat exchanger

### 1. INTRODUCTION

Time delay is a typical phenomenon in real processes that is usually caused by information, mass or energy transport. It can be also caused by mass or energy accumulation in dynamic systems connected in series. Typical time-delay processes in chemical industry are tubular heat exchangers. There are several approaches to control heat exchangers as time-delay systems and the Smith predictor and its modifications belong to the approaches offering good results. The predictor based controllers are known as time delay compensators and they have been applied in many engineering fields, mainly in the process industry (Huzmezan et al. (2002), Normey-Rico et al. (1997)), but also in robotics (Normey-Rico and Camacho (1999)) and internet connection (Mascolo (2006)).

The Smith predictor and its modifications (Šulc and Vítečková (2004)) can be successfully used for control of processes with significant time delay, when the model of the controlled system and the model of the time delay are very well known. The modifications are used to improve the closed-loop control responses of time-delay integrating systems, time-delay unstable systems, time-delay systems with disturbances, etc. (Dostál et al. (2008)).

The paper presents using a modified Smith predictor for control of a co-current tubular heat exchanger, in which the kerosene is heated by hot water. Kerosene flows in the inner tube and water flows in the outer tube. The operation of the heat exchanger is affected by disturbances that are represented by changes of the kerosene inlet temperature. The objective is to heat the outlet temperature of the kerosene to the demanded value by the mass flow of heating water. The heat exchanger represents a non-linear system with time delay.

### 2. SMITH PREDICTOR

One of the most popular time delay compensating method is the Smith predictor. The structure of the Smith predictor is shown in Figure 1. This structure can be divided into two parts. The first part is the primary controller  $G_r(s)$ , which is usually the PID controller and the second part is the predictor structure. The predictor is composed of the plant model without time delay  $G_m(s)$  and of the model of the time delay  $e^{-D_m s}$ . The complete process model is  $P_m(s) = G_m(s)e^{-D_m s}$ . The model  $G_m(s)$  is used to compute an open loop prediction. The controller  $G_r(s)$  can be tuned for the plant model without time delay, when there are no model errors or disturbances and the error between process output and model output is zero. For successful modelling, following three characteristics of the Smith predictor have to be analysed:  $P(s) = P_m(s)$ ,  $G(s) = G_m(s)$ ,  $D = D_m$ . The Smith predictor structure has for the nominal case (no modelling errors) these fundamental properties (Normey-Rico and Camacho (2008)):

- time delay compensation and prediction
- performance limitation of the Smith predictor

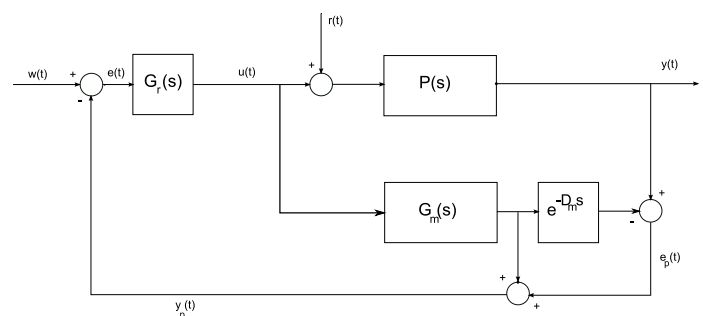


Fig. 1. Smith predictor

2.1 Property 1: Time delay compensation and prediction

It is easy to see in Figure 1, the error signal  $e_p(t)$  is zero, if  $r(t) = 0$  and  $G(s)e^{-Ds} = G_m(s)e^{-D_m s}$ . The characteristic equation is

$$1 + G_r(s)G_m(s) = 0 \quad (1)$$

Compare the equation (1) to the time-delay dependent one obtained in the PID case

$$1 + G_r(s)G_m(s)e^{-D_m s} = 0 \quad (2)$$

where the extra phase introduced by the time delay reduces the phase margin. The feedback signal  $y_p(t)$  anticipates the system output for changes in the set point, although this is not the case for disturbances

$$y_p(t) = y(t + D_m) + P_m(s) [r(t) - r(t + D_m)] \quad (3)$$

For slow changes of the disturbance, it is a good prediction of  $y(t + D_m)$ . But if the disturbance changes rapidly then it cannot be eliminated from the feedback signal  $y_p(t)$  (Normey-Rico and Camacho (2008)).

2.2 Property 2: Performance limitation for the Smith predictor

The structure of the Smith predictor divides the plant into two parts. The first is invertible  $G_m(s)$  and the second is non-invertible  $e^{-D_m s}$ . Using this idea and considering that ideal controller with infinity gain could be applied, it follows (Fig. 2)

$$G'_r(s) = \frac{G_r(s)}{1 + G_r(s)G_m(s)} = (G_m(s))^{-1} \quad (4)$$

The ideal transfer function between the reference and the output is a simple delay. In real conditions the ideal controller cannot be applied. Even in the ideal case, if a disturbance is applied at  $t = 0$ , it is necessary to wait until  $t = 2D_m$  to note the effect of the controller on the output (Normey-Rico and Camacho (2008)).

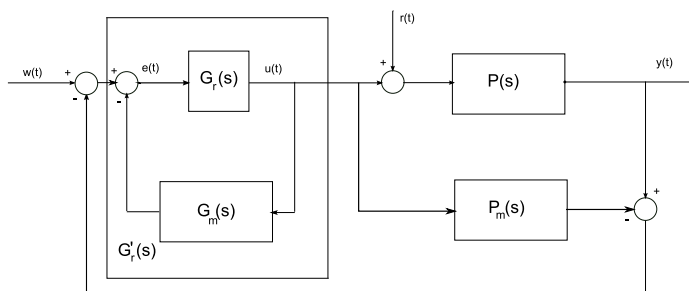


Fig. 2. Equivalent control structure of the Smith predictor

3. MODIFICATION OF THE SMITH PREDICTOR FOR DISTURBANCE COMPENSATION

A modification of the Smith predictor with feed-forward control loop can be used for improving the closed-loop control response when the controlled system is affected by measurable disturbances. When the disturbance is not measurable, this approach can be applied, but the disturbance has to be estimated (Normey-Rico and Camacho

(2008)). Figure 3 shows a block diagram of the Smith predictor for disturbance compensation, where  $P_{mr}(s)$  represents the model for  $P_r(s)$ .  $P_r(s)$  represents the model of the disturbance dynamics (Normey-Rico and Camacho (2008)).

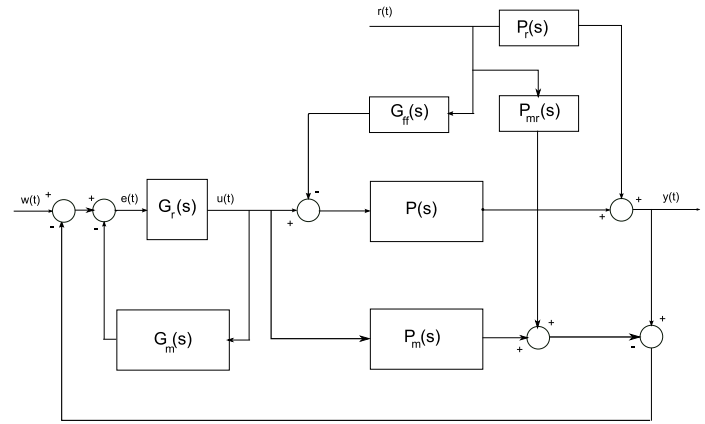


Fig. 3. Smith predictor modification for disturbance compensation

In the ideal case, when  $P_{mr}(s) = P_r(s)$  and  $P_m(s) = P(s)$ , the transfer function  $\frac{Y(s)}{R(s)}$  is in the form

$$\frac{Y(s)}{R(s)} = [P_r(s) - G_{ff}(s)P(s)] \quad (5)$$

The disturbance effect can be eliminated from the output of the process independently on the type of disturbance if exists such  $G_{ff}(s)$  that

$$G_{ff} = \frac{P_r(s)}{P(s)} \quad (6)$$

Consider the plant and the load disturbance transfer functions  $P(s)$  and  $P_r(s)$  defined as  $P(s) = G(s)e^{-D_s}$ ,  $P_r(s) = G_{rr}(s)e^{-D_r s}$ . Two situations can occur (Normey-Rico and Camacho (2008)):

- $D < D_r$

In this case, the controller is in the form

$$G_{ff} = \frac{G_r(s)}{G(s)} e^{-(D_r - D)s} \quad (7)$$

If  $\frac{G_r(s)}{G(s)}$  can be computed the disturbance is eliminated from the output. Otherwise, a pseudo inverse of  $G(s)$  can be computed  $G_{ff}(s)P(s) = P(s)X(s)$ . The final  $\frac{Y(s)}{R(s)}$  is

$$\frac{Y(s)}{R(s)} = e^{-D_r s} G_r(s) [1 - X(s)], \quad (8)$$

where  $1 - X(s)$  has zero static gain and the fastest achievable response (Normey-Rico and Camacho (2008)).

- $D > D_r$

In this case, it is not possible to compute the inverse of  $e^{(D_r - D)s}$ . The feed-forward controller is given by

$$G_{ff} = \frac{G_r(s)}{G(s)} \quad (9)$$

Table 1. Heat exchanger parameters and inputs

Variable / Unit	Value	Variable / Unit	Value
$l$ / m	2	$\rho_1$ / kg m <sup>-3</sup>	810
$D_3$ / m	0.05	$\rho_2$ / kg m <sup>-3</sup>	8930
$D_2$ / m	0.028	$\rho_3$ / kg m <sup>-3</sup>	1000
$D_1$ / m	0.025	$C_{P1}$ / kJ kg <sup>-1</sup> K <sup>-1</sup>	2100
$\alpha_1^s$ / W m <sup>-2</sup> K <sup>-1</sup>	750	$C_{P2}$ / kJ kg <sup>-1</sup> K <sup>-1</sup>	385
$\alpha_2^s$ / W m <sup>-2</sup> K <sup>-1</sup>	1480	$C_{P3}$ / kJ kg <sup>-1</sup> K <sup>-1</sup>	4186
$\dot{m}_1^s$ / kg s <sup>-1</sup>	0.0556	$\vartheta_{1in}^s$ / °C	20
$\dot{m}_{3in}^s$ / kg s <sup>-1</sup>	0.0417	$\vartheta_{3in}^s$ / °C	85

and the final transfer function is

$$\frac{Y(s)}{R(s)} = e^{-D_r(s)} G_r(s) \left[ 1 - X(s)e^{-(D-D_r)s} \right] \quad (10)$$

Note that even in this case the solution is better than the one obtained when the feed forward is not used. The advantage of this solution is less important when  $D_r \rightarrow 0$ . The previous structure cannot be used when disturbance is not measurable. Using an estimation of disturbance  $r(t)$ , the idea can be used to improve the controller. One advantage of this approach is that the controller can be easily tuned to reject other types of disturbances and not only step ones (Normey-Rico and Camacho (2008)).

#### 4. MODEL OF THE TUBULAR HEAT EXCHANGER

The controlled process is a co-current tubular heat exchanger, in which kerosene is heated by hot water. Kerosene flows in the inner copper tube and water flows in the outer copper tube. The operation of the heat exchanger is affected by the disturbance that is represented by changes of the kerosene inlet temperature. The objective is to heat the outlet temperature of the kerosene to the demanded value by the mass flow of heating water. The heat exchanger represents a non-linear system with variable time delay, where the controlled output is the outlet kerosene temperature and the control input is the mass flow-rate of heating water.

Technological parameters and steady-state inputs of the heat exchanger are listed in the Table 1, where  $l$  is the length of the heat exchanger,  $D$  is the tube diameter,  $\rho$  is the density,  $\alpha$  is the heat transfer coefficient,  $C_P$  is the specific heat capacity and  $\dot{m}$  is the mass flow rate. The subscripts have following meaning: 1–kerosene or from the copper tube to kerosene or the inner diameter of the inner tube, 2–copper or from water to the copper tube or the outer diameter of the inner tube, 3–water or the inner diameter of the outer tube and  $in$  – the inlet. The superscript  $s$  represents the steady-state.

The mathematical model of the tubular heat exchanger is represented by three nonlinear partial differential equations in the form

$$T_1 \frac{\partial \vartheta_1(z, t)}{\partial t} + T_1 w_1 \frac{\partial \vartheta_1(z, t)}{\partial z} = -\vartheta_1(z, t) + \vartheta_2(z, t) \quad (11)$$

$$T_2 \frac{\partial \vartheta_2(z, t)}{\partial t} = Z_1 \vartheta_1(z, t) - \vartheta_2(z, t) + Z_2 \vartheta_3(z, t) \quad (12)$$

$$T_3 \frac{\partial \vartheta_3(z, t)}{\partial t} + T_3 w_3(z, t) \frac{\partial \vartheta_3(z, t)}{\partial z} = -\vartheta_3(z, t) + \vartheta_2(z, t) \quad (13)$$

where

$$T_1 = \frac{D_1 \rho_1 C_{P1}}{4\alpha_1}, \quad T_2 = \frac{(D_2^2 - D_1^2) \rho_2 C_{P2}}{4(D_1 \alpha_1 + D_2 \alpha_2)},$$

$$T_3 = \frac{(D_3^2 - D_2^2) \rho_3 C_{P3}}{4D_2 \alpha_2}$$

$$w_1 = \frac{q_1}{\pi D_1^2}, \quad w_3(z, t) = \frac{q_3(z, t)}{\pi(D_3^2 - D_2^2)}$$

$$q_3(z, t) = \frac{\dot{m}_3(z, t)}{\rho_3}$$

$$Z_1 = \frac{D_1 \alpha_1}{D_1 \alpha_1 + D_2 \alpha_2}, \quad Z_2 = \frac{D_2 \alpha_2}{D_1 \alpha_1 + D_2 \alpha_2}$$

For simulations purposes, the heat exchanger was split in ten sections, each of them represented by three ordinary nonlinear differential equations with delayed inputs. The model was generated using MATLAB–Simulink environment.

For control purposes, the properties of the heat exchanger have been examined by simulation experiments. The model of the heat exchanger was identified using the Strejc method (Mikleš and Fikar (2007)) in the form of the transfer function

$$P_m(s) = \frac{K}{(Ts + 1)^n} e^{-Ds} \quad (14)$$

where  $n$  is the order of the system,  $K$  is the gain,  $T$  is the time constant and  $D$  is the time delay.

For the identification, following step changes of the inlet mass flow-rate of heating water were generated at the time  $t = 0$  s:  $\pm 10\%$ ,  $\pm 20\%$ ,  $\pm 30\%$ . Step responses of the outlet kerosene temperature on the generated inlet step changes are shown in Figure 4. According to these step changes, the heat exchanger is a time-delay nonlinear system with asymmetric dynamics.

The heat exchanger was identified in the form of the 3rd order plus time delay system (Table 2). For various step responses, we obtained intervals for values of the gain  $K$ , the time constant  $T$  and the time delay  $D$ .

Table 2. Identification of the process dynamics

$n = 3$	$K_{min}$	$K_{max}$	$T_{min}$	$T_{max}$	$D_{min}$	$D_{max}$
	0.055	0.071	11.382	19.241	14.422	22.844

It is supposed further that the dynamics of the heat exchanger is affected by disturbances. The disturbances are caused by changes of the kerosene inlet temperature. The model of the disturbance dynamics was identified using the Strejc method. The generated step changes of the inlet kerosene temperature were  $\pm 2^\circ\text{C}$ . The step responses of outlet kerosene temperature are depicted in the Figure 5. The values of the identified parameters are summarized in Table 3, where  $K_r$  represents the gain,  $T_r$  is the time constant and  $D_r$  is the time delay of the model of the disturbance dynamics.

Table 3. Identified parameters of the disturbance dynamics

$n_r = 2$	$K_r$	$T_r$	$D_r$
	0.6007	4.7566	20.8980

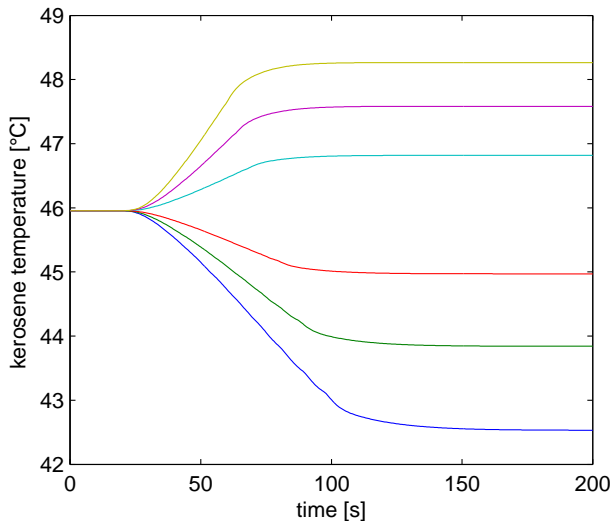


Fig. 4. Step responses of the outlet kerosene temperature on the step changes of the control input, where input change +10% is represented by cyan line, -10% is represented by red line, +20% is represented by magenta line, -20% is represented by green line, +30% is represented by yellow line, -30% is represented by blue line

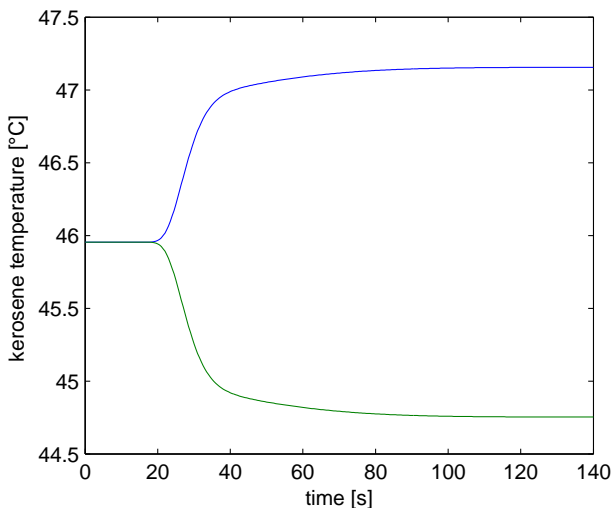


Fig. 5. Step responses of the outlet kerosene temperature on the step changes of the disturbance, where disturbance change +2% is represented by blue line, -2% is represented by green line

### 5. CONTROL OF THE TUBULAR HEAT EXCHANGER

For the third order model of the heat exchanger, four PI controllers were designed ( $G_{R1} - G_{R4}$ ). The transfer function of the PI controller is in the form

$$G_R(s) = Z_R + \frac{Z_R}{T_I s} \quad (15)$$

where  $Z_R$  is the gain and  $T_I$  is the reset time of the controller (Bakošová et al. (2003)).

The controllers  $G_{R1} - G_{R3}$  were tuned using the experimental methods. The controller  $G_{R1}$  was designed for the

model described by maximal values of identified parameters, the controller  $G_{R2}$  was designed for the model described by the minimal values of identified parameters and the controller  $G_{R3}$  was designed for the model described by the mean (nominal) values of parameters (Table 2). Because the heat exchanger can be represented also as a system with interval parametric uncertainty, a robust PI controller  $G_{R4}$  was tuned using the method described in Závacká et al. (2007). The parameters of designed controllers are listed in Table 4.

Table 4. Parameters of set-point tracking performances using the Smith predictor

controller	model parameters	$Z_R$	$T_I$	IAE
$G_{R1}$	<i>maximal</i>	8.44	49.14	1794
$G_{R2}$	<i>minimal</i>	11.78	83.07	2035
$G_{R3}$	<i>nominal</i>	9.98	25.52	4737
$G_{R4}$	<i>interval</i>	5.00	25.00	2150

Control of the heat exchanger without disturbance using the Smith predictor was simulated at first and all designed controllers were used in the predictor structure. The step change of the set-point was done at time  $t = 1500$  s from  $50^\circ\text{C}$  to  $40^\circ\text{C}$ . The closed-loop control responses obtained using four designed controllers are shown in Figure 6, where the closed-loop control using  $G_{R1}$  is represented by the solid magenta line, using  $G_{R2}$  is represented by the cyan dash-dot line, using  $G_{R3}$  is represented by the red dotted line, using  $G_{R4}$  is represented by the green dashed line. The quality of the closed-loop control was evaluated using IAE (Mikleš and Fikar (2007)) quality criteria (16)

$$\text{IAE} = \int_0^\infty |e(t)| dt \quad (16)$$

Obtained values of IAE are enumerated in Table 4. The best value of the IAE was reached using controller  $G_{R1}$ .

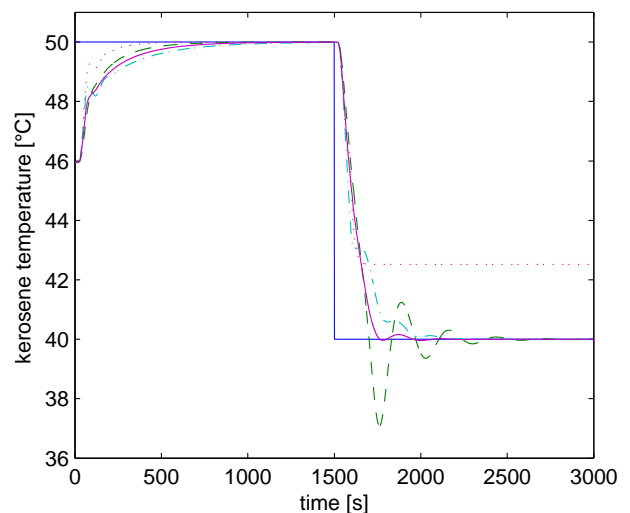


Fig. 6. Set-point tracking assured using  $G_{R1}$  (solid magenta line),  $G_{R2}$  (cyan dash-dot line),  $G_{R3}$  represented by the (red dotted line) and  $G_{R4}$  (green dashed line)

Then the control of the heat exchanger affected by disturbances was analyzed. The disturbance is represented by

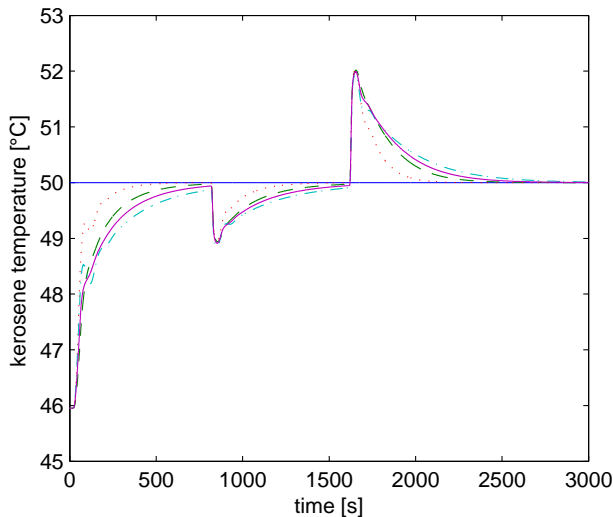


Fig. 7. Disturbance rejection without disturbance compensation assured using  $G_{R1}$  (solid magenta line),  $G_{R2}$  (cyan dash-dot line),  $G_{R3}$  represented by the (red dotted line) and  $G_{R4}$  (green dashed line)

the step change of the kerosene inlet temperature. This temperature decreased in  $2^{\circ}\text{C}$  at the time  $t = 800\text{s}$  and then the inlet kerosene temperature increased in  $2^{\circ}\text{C}$  at the time  $t = 1600\text{s}$ . Table 5 contains the controllers and the associated calculated values of IAE. The minimal value has been reached using  $G_{R3}$  controller. Figure 7 shows the closed-loop control responses obtained using the Smith predictor without disturbance compensation, where the closed-loop control using  $G_{R1}$  is represented by the solid magenta line, using  $G_{R2}$  is represented by the cyan dash-dot line, using  $G_{R3}$  is represented by the red dotted line, using  $G_{R4}$  is represented by the green dashed line.

Table 5. Parameters of disturbance rejection performances using the Smith predictor

controller	model parameters	$Z_R$	$T_I$	IAE
$G_{R1}$	<i>maximal</i>	8.44	49.14	1371
$G_{R2}$	<i>minimal</i>	11.78	83.07	1533
$G_{R3}$	<i>nominal</i>	9.98	25.52	740
$G_{R4}$	<i>interval</i>	5.00	25.00	1210

Then the modified Smith predictor with feed-forward disturbance compensation was applied. In Table 6, we can see the used controllers and the associated calculated values of IAE. The minimal value of IAE was reached also using  $G_{R3}$  controller. As can be seen in Table 6, the control performances generated by the controllers  $G_{R1}$  and  $G_{R3}$  lead to the higher values of IAE in comparison to the Smith predictor without disturbance compensation (Table 5). Figure 8 shows the closed-loop control response obtained using the modified Smith predictor with disturbance compensation, where the closed-loop control using  $G_{R1}$  is represented by the solid magenta line, using  $G_{R2}$  is represented by the cyan dash-dot line, using  $G_{R3}$  is represented by the red dotted line, using  $G_{R4}$  is represented by the green dashed line.

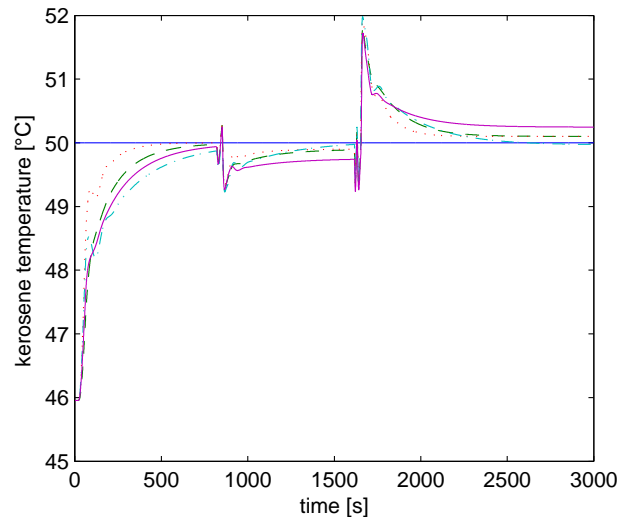


Fig. 8. Disturbance rejection with disturbance compensation assured using  $G_{R1}$  (solid magenta line),  $G_{R2}$  (cyan dash-dot line),  $G_{R3}$  represented by the (red dotted line) and  $G_{R4}$  (green dashed line)

## 6. CONCLUSION

The possibility to use the Smith predictor and the modified Smith predictor with feed-forward disturbance compensation for control of a time-delay system was studied in this paper. The controlled system was a tubular heat exchanger, which was a nonlinear time delay system. As the process was identified as a system with interval parametric uncertainty, the four PI controllers were designed for the Smith predictor and modified Smith predictor control structure. The three controllers were designed using experimental methods. They were tuned using the maximal, minimal and nominal process model parameters. The fourth PI controller was designed using robust control approach. Because of complicated dynamics of the controlled process, obtained simulation results are difficult to compare. But it can be stated, that using the robust controller in both control structures never led to the worst control response. Using the robust controller in the modified Smith predictor lead to the better disturbance compensation in comparison to the Smith predictor without disturbance compensation. Using the controller designed for the nominal values of process model parameters gave the best results in the task of disturbance rejection, but this controller led to the worst result in the task of set-point tracking.

In the next work the heat exchanger with counter-current of cooling medium will be studied. The obtained results will be compared to the results obtained using the heat exchanger with co-current of cooling medium. The studied

Table 6. Parameters of disturbance rejection performances using modified the Smith predictor

controller	model parameters	$Z_R$	$T_I$	IAE
$G_{R1}$	<i>maximal</i>	8.44	49.14	1422
$G_{R2}$	<i>minimal</i>	11.78	83.07	1198
$G_{R3}$	<i>nominal</i>	9.98	25.52	790
$G_{R4}$	<i>interval</i>	5.00	25.00	1116

control methods will be applied to the control of a real model of the heat exchanger.

#### ACKNOWLEDGMENTS

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