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ON MODELING AND IDENTIFICATION OF SYSTEMS WITH GENERAL BACKLASH

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Abstract: The notion of general backlash is introduced where instead of straight lines determining the upward and downward parts of backlash characteristic general curves are considered. An analytic form of general backlash characteristic description is proposed, which is based on appropriate switching and internal functions. Hence the multi-valued mapping is represented by one equation. All the parameters in the model equation describing this hard nonlinearity are separated; consequently their estimation can be solved as a quasi-linear problem using an iterative parameter estimation method with internal variable estimation.

Keywords: Backlash, general backlash, modeling, identification.

1 INTRODUCTION

One of the most important nonlinearities that limit control systems performance in many applications is the so-called backlash (Kalas et al, 1985). The backlash can be classified as a hard (i.e. non-differentiable) and dynamic nonlinearity. It is well known that this kind of nonlinearity may often cause delays, oscillations and inaccuracy which severely limit the performance of control systems, therefore, the identification of systems with unknown backlash is an open theoretical problem of major relevance to control applications.

In control systems it is assumed that the backlash is “linear”, i.e., straight lines approximate the upward and downward curves of the characteristic. This simplifies the system description, however, in some cases it may lead to inaccuracies. Therefore it may be appropriate to generalize the backlash and consider general upward and downward curves.

In this paper, an analytic description of this hard dynamic nonlinearity is introduced, which uses appropriate switching functions and internal variables. This is based on the recently proposed description of linear backlash (Vörös, 2008), where the multi-valued characteristic is represented by one equation. The general backlash parameters in the model equation are separated; hence their estimation can be solved as a quasi-linear problem using an iterative method with internal variable estimation similarly as in (Vörös, 1999, 2002).

2 BACKLASH

The discrete-time mathematical model for the standard backlash nonlinearity with inputs u(t) and outputs x(t) shown in Fig. 1, is given by (Tao and Kokotovic, 1993), (Cerone and Regruto, 2007)

$$x(t)=\begin{cases} m_L[u(t)+c_L] & u(t)\le z_L \\ m_R[u(t)-c_R] & u(t)\ge z_R \\ x(t-1) & z_L\le u(t)\le z_R \end{cases} \tag{1}$$

with z_L and z_R defined by

$$x(t-1) = m_L(z_L + c_L) \quad (2)$$

$$x(t-1) = m_R(z_R - c_R) \quad (3)$$

where the slopes m_L, m_R, the dead zone constants $c_L > 0$, $c_R > 0$ characterize the backlash and z_L and z_R are the u-axis values of intersections of the two lines with the horizontal inner segment containing $x(t-1)$.

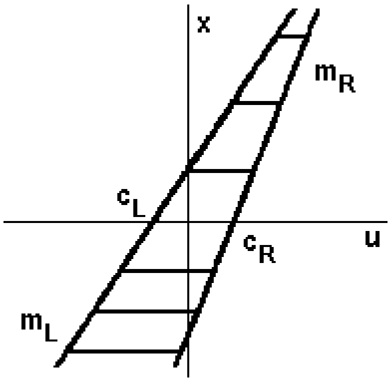

Fig. 1 Backlash characteristic

A special form of backlash description was proposed in (Vörös, 2008) to specify the three branches of (1) in one equation. This is based on the function

$$h(s) = \begin{cases} 0, & if\ s > 0 \\ 1, & if\ s \le 0 \end{cases} \quad (4)$$

switching between two sets of values, i.e., $(-\infty, s)$ and (s, ∞), and the complementary function to $h(s)$, that is $[1 - h(s)]$. Defining the following variables based on (2) and (3):

$$\begin{aligned} f_1(t) &= h[u(t) - z_L] \\ &= h\{[m_L u(t) + m_L c_L - x(t-1)] / m_L\} \end{aligned} \quad (5)$$

$$\begin{aligned} f_2(t) &= h[z_R - u(t)] \\ &= h\{[x(t-1) - m_R u(t) + m_R c_R] / m_R\} \end{aligned} \quad (6)$$

the backlash, which is a multi-valued mapping, can be modeled by one difference equation as:

$$\begin{aligned} x(t) &= m_L u(t) f_1(t) + m_L c_L f_1(t) + m_R u(t) f_2(t) \\ &- m_R c_R f_2(t) + x(t-1)[1 - f_1(t)][1 - f_2(t)] \end{aligned} . \quad (7)$$

The input/output relation (7) is identical with that of (1). The slopes of straight lines m_L and m_R may be simultaneously positive or negative, while the constants c_L and c_R, determining the dead zone, must be positive. This model allows the upward and downward line slopes to be different provided that the intersection of the two lines is not in the region of practical interest.

3 GENERAL BACKLASH

In the above mentioned case of "linear" backlash the left and right branches of the characteristic are considered to be straight lines. However, in some applications the straight lines are only advantageous approximations of general curves constituting the left and right branches of backlash as shown in Fig. 2. Therefore the backlash can be generalized in the following way.

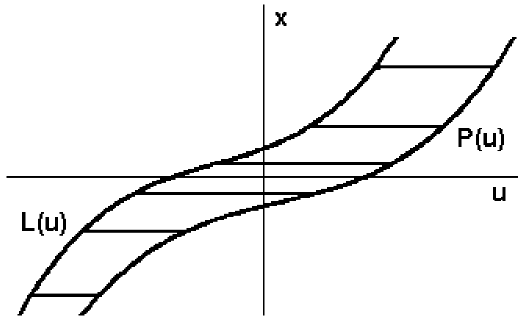

Fig. 2 General backlash characteristic

The general backlash characteristic can be described by the equation

$$x(t) = \begin{cases} L[u(t)] & u(t) \le z_L \\ R[u(t)] & u(t) \ge z_R \\ x(t-1) & z_L \le u(t) \le z_R \end{cases} \quad (8)$$

where the mappings $L[u(t)]$ and $R[u(t)]$ describe the left and right branches of the characteristic, respectively, and the u-axis values z_L and z_R, by analogy with (2) and (3), are given as follows:

$$x(t-1) = L(z_L) \quad (9)$$

$$x(t-1) = R(z_R) \quad (10)$$

Assume the left and right curves can be approximated by the polynomials

$$L[u(t)] = \sum_{i=1}^{n} m_{Li}[u(t) + c_L]^i \quad (11)$$

$$R[u(t)] = \sum_{i=1}^{n} m_{Ri}[u(t) - c_R]^i \quad (12)$$

respectively, where $c_L > 0$, $c_R > 0$ are the intersections of $L[u(t)]$ and $R[u(t)]$ with the u-axis. For ease of explanation assume n = 3. Then the general backlash characteristic can be written as

$$x(t) = \begin{cases} m_{L1}[u(t)+c_L] + m_{L2}[u(t)+c_L]^2 \\ + m_{L3}[u(t)+c_L]^3 & u(t) \le z_L \\ m_{R1}[u(t)-c_R] + m_{R2}[u(t)-c_R]^2 \\ + m_{R3}[u(t)-c_R]^3 & u(t) \ge z_R \\ x(t-1) & z_L \le u(t) \le z_R \end{cases} \tag{13}$$

where

$$x(t-1) = m_{L1}[z_L(t)+c_L] + m_{L2}[z_L(t)+c_L]^2 + m_{L3}[z_L(t)+c_L]^3 \tag{14}$$

$$x(t-1) = m_{R1}[z_R(t)+c_R] + m_{R2}[z_R(t)+c_R]^2 + m_{R3}[z_R(t)+c_R]^3 \tag{15}$$

Now, an analogous approach, as was done in the previous Section, can be applied to the description of general backlash. After introducing the internal variables

$$\xi_1(t) = u(t) + c_L \tag{16}$$

$$\xi_2(t) = u(t) - c_R \tag{17}$$

the following variables based on (14) and (15) can be defined:

$$f_1(t) = h[\xi_1(t)] = h[m_{L1}\xi_1(t) + m_{L2}\xi_1^2(t) + m_{L3}\xi_1^3(t) - x(t-1)] \tag{18}$$

$$f_2(t) = h[\xi_2(t)] = h[x(t-1) - m_{R1}\xi_2(t) - m_{R2}\xi_2^2(t) - m_{R3}\xi_2^3(t)] \tag{19}$$

Then the general backlash can be modeled by one difference equation as follows:

$$\begin{aligned} x(t) &= m_{L1}\xi_1(t)f_1(t) + m_{L2}\xi_1(t)^2 f_1(t) \\ &+ m_{L3}\xi_1(t)^3 f_1(t) + m_{R1}\xi_2(t)f_2(t) \\ &+ m_{R2}\xi_2(t)^2 f_2(t) + m_{R3}\xi_2(t)^3 f_2(t) \\ &+ x(t-1)[1-f_1(t)][1-f_2(t)] \end{aligned} \tag{20}$$

To include the deadzone parameters c_L and c_R into the backlash model, we can separate the first $\xi_1(t)$ and $\xi_2(t)$ in (20) and half-substitute from (16) and (17)

$$\begin{aligned} x(t) &= m_{L1}u(t)f_1(t) + m_{L1}c_L f_1(t) \\ &+ m_{L2}\xi_1(t)^2 f_1(t) + m_{L3}\xi_1(t)^3 f_1(t) \\ &+ m_{R1}u(t)f_2(t) - m_{R1}c_R f_2(t) \\ &+ m_{R2}\xi_2(t)^2 f_2(t) + m_{R3}\xi_2(t)^3 f_2(t) \\ &+ x(t-1)[1-f_1(t)][1-f_2(t)] \end{aligned} \tag{21}$$

Now the input/output relation for the generalized backlash (21) is identical with that of (8). All the model parameters are separated and the model is linear in the input, output and internal variables. This model allows the upward and downward curves to be different provided that the intersection of the two curves is not in the region of practical interest.

4 PARAMETER ESTIMATION

The proposed new model can be used for estimation of generalized backlash parameters. Defining the following vector of data

$$\begin{aligned} \varphi(t) = [&u(t)f_1(t), f_1(t), \xi_1(t)^2 f_1(t), \xi_1(t)^3 f_1(t), \\ &u(t)f_2(t), -f_2(t), \xi_2(t)^2 f_2(t), \xi_2(t)^3 f_2(t)]^T \end{aligned} \tag{22}$$

and the vector of parameters

$$\begin{aligned} \theta = [&m_{L1}, m_{L1}c_L, m_{L2}, m_{L3}, \\ &m_{R1}, m_{R1}c_R, m_{R2}, m_{R3}]^T \end{aligned} \tag{23}$$

the generalized backlash model can be written in the vector form

$$x(t) - x(t-1)[1-f_1(t)][1-f_2(t)] = \varphi(t)\,\theta \tag{24}$$

As the variables $\xi_1(t)$, $\xi_2(t)$, $f_1(t)$ and $f_2(t)$ in (22) are unmeasurable and must be estimated, an iterative parameter estimation process can be applied similarly as in (Vörös, 1999, 2002).

5 EXAMPLE

The method for the identification of general backlash was implemented and tested in MATLAB. Several cases were simulated and the estimations of parameters were carried out on the basis of input and output records. The performance of the proposed methods is illustrated on the following example.

The symmetric general backlash shown in Fig. 3 was simulated with the following parameters: $m_{L1} = 0.5$, $m_{L2} = 0.0$, $m_{L3} = 0.3$, $c_L = 0.7$, $m_{R1} = 0.5$, $m_{R2} = 0.0$, $m_{R3} = 0.3$, $c_R = 0.7$. The identification was performed on the basis of 800 samples of uniformly distributed random inputs with $|u(t)| < 2.0$ and simulated outputs. Normally distributed random noise with zero mean and signal-to-noise ratio - SNR = 25 (the square root of the ratio of output and noise variances) was added to the outputs. The iterative estimation algorithm was applied with initial values $m_{L1} = m_{R1} = 1$ and $c_L = c_R = 0.001$ for the first estimates of $\xi_1(t)$, $\xi_2(t)$, $f_1(t)$ and $f_2(t)$. The process of parameter estimation is shown in Fig. 4 (the top-down order of parameters is $c_R = c_L$, $m_{R1} = m_{L1}$, $m_{R3} = m_{L3}$, $m_{R2} = m_{L2}$). The estimates converge to the values of real parameters after 5 iterations.

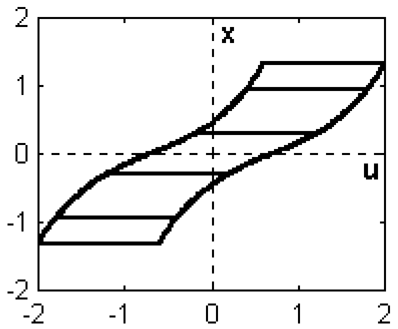

Fig. 3 Symmetric general backlash characteristic

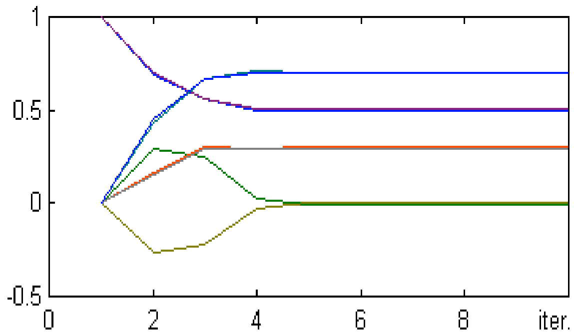

Fig. 4 Parameter estimates

6 CONCLUSIONS

In this paper a new analytic form of general backlash characteristic was introduced, where this three-branch nonlinearity is described by one output equation with separated parameters. As all the parameters in the resulting difference equation are separated their estimation can be performed by an appropriate iterative algorithm with internal variable estimation.

Finally note that in many real control systems the backlash appears in a cascade connection with a linear dynamic system. The possible cases are the cascade systems where the general backlash is preceded or followed by a linear dynamic system. The proposed form of general backlash description can be easily incorporated into such cascade systems and used for their identification.

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