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PREDICTIVE FUNCTIONAL CONTROL OF THERMAL PROCESS WITH DEAD TIME

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Abstract: Paper deals with mathematical model of thermal process and its control by two different controllers. Firstly, the real system and mathematical model is briefly described. Two different controllers are designed afterwards. First of them is PID controller designed in empirical way by using Åström-Hägglund method. The second one is simple predictive controller Predictive Functional Controller based on Richalet's principal.

Keywords: Thermal process, control, PFC, Åström-Hägglund method

1 INTRODUCTION

Thermal process with dead-time was developed at Tomas Bata University in Zlin within the project GA 102/03/0625 Konsorciální přístup k vývoji laboratorních modelů (Klán et al., 2005). First principle model of the process was derived and two controllers (PID and PFC) were designed and applied.

2 THERMAL PROCESS

Thermal process with dead-time (see Fig. 1) is based on the heat transport principle. Heat transfer medium is water. It flows through the pump, which is powered by the voltage 0-10 V. Water is pumped to the boiler (maximum power is 800 W), then to the fifteen meters long heat-insulated tube, which causes the dead-time. The last apparatus in the process is a heat consumer. In our case it is a heat exchanger water-air. It is possible to change the heat consumption by two electric fans. The first is controlled continuously by the voltage 0-10 V, the second is manipulated only by switching on or off.

Water temperature is measured by three platinum thermometers. The first is behind the boiler, the second is behind the heat-insulated tube and the third is behind the exchanger.

The connection to the computer is provided by the CTRL V3 unit. This unit has four analog inputs, two analog outputs, in the range 0-10 V, and four logical inputs and outputs. Connection between CTRL and computer is provided by RS 232 bus.

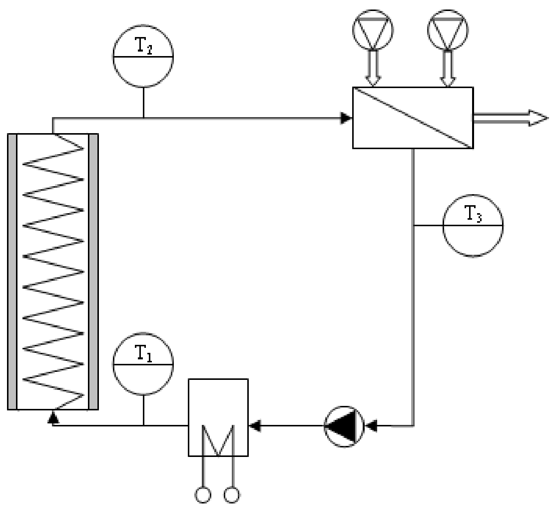

Figure 1 – Thermal process with dead time chart

At first it was necessary to modify the process. It was impossible to get steady states (temperature of the water flowing from the heat exchanger to the boiler increased continuously, because the heat exchanger has not enough power to cool down the water). The tube between the exchanger and the water pump was equipped with a valve system. These valves broke the water closed loop and enabled to pump the water from a water reservoir and to flow out to a sink.

3 MATHEMATICAL MODEL

Mathematical model of the thermal process consists of four parts. Mathematical equations describing water pump, boiler, heat-insulated tube and heat exchanger were derived, then the unknown parameters were analytically or numerically estimated.

Because we are not interested in variables along device dimension we can consider process behavior as a lumped system. Only water capacity is taken into account for simplicity.

3.1 Water pump model

The water pump is the first part of the process, which was necessary to describe. The flow rate is controlled by the voltage 0-10 V. Only static characteristic was used for pump description and its dynamics was omitted comparing to whole thermal process dynamics. An approximation of static pump behavior was used in the form

$$Q = r.(u + t)^{s} \tag{1}$$

where Q is flow rate (l/min), u is voltage (V) and r, s, t are pump parameters.

3.2 Boiler model

Energy balance equation was considered for the boiler in a form

$$M.c.T_{IN} + P = K_O.(T_O - T_{OK}) + M.c.T_O + G_O.c.\frac{dT_O}{dt} \tag{2}$$

where

M is mass flow rate ($kg.s^{-1}$),
c is water thermal capacity ($J.kg^{-1}.K^{-1}$),
P is boiler power (W),
G_o is hold-up coefficient (kg),
K_o is heat transfer coefficient ($W.K^{-1}$),
T_o is water temperature on boiler output (°C),
T_{IN} is water temperature on boiler input (°C) and
T_{OK} is ambient temperature.

If we omit the last term (the accumulation term) in (2) we get steady-state description of the boiler behavior.

Unknown parameters are heat transfer coefficient K_0 and boiler hold-up G_0.

3.3 Heat-insulated tube model

Heat-insulted tube is the next part of the process. Fifteen meters long tube causes the transport delay of the process. Ideal transport delay is described by

$$d = \frac{S.l}{Q} \tag{3}$$

where d is transport delay (s), S is tube cross-section (m^2), l is tube length (m) and Q is volumetric water flow rate (m^3/s).

Real process is a system with distributed parameters (with heat losses and thermal conductivity). Such a system should be described by partial differential equations. But we are not interested in temperatures along the tube. We can approximate process behavior by lumped system and a time delay.

$$M.c.T_{O(t-d)} = K_S.(T_S - T_{OK}) + M.c.T_S + G_S.c.\frac{dT_S}{dt} \tag{4}$$

where

G_S is tube hold-up (kg),
K_S is heat transfer coefficient ($W.K^{-1}$),
T_o is water temperature on tube input (°C) and
T_S is water temperature on tube output (°C)

Unknown parameters are heat transfer coefficient K_S and hold up G_S.

3.4 Heat exchanger model

Heat exchanger is the last part of the process to model. Exchanger fans are off for the case of simplicity. Mass balance equation for the heat exchanger is

$$M.c.T_S = K_V.(T_V - T_{OK}) + M.c.T_V + G_V.c.\frac{dT_V}{dt} \tag{5}$$

where

G_V is exchanger hold-up (kg),
K_V is heat transfer coefficient ($W.K^{-1}$),
T_S is water temperature on exchanger input (°C) and
T_V is water temperature on exchanger output (°C)

Finally all unknown parameters were estimated by using optimization methods from measured data and the model was realized in Simulink and verified, (Mareš et all., 2008).

4 CONTROL EXPERIMENTS

Two controllers were chosen for the process model control. The first one is PID controller with parameters set according to Åström-Hägglund method and the second one is simple Predictive Functional Controller (PFC).

Process model is controlled as a system with dead time. Controlled variable is output temperature from heat exchanger and manipulated variable is a boiler power.

4.1 *Aström-Hägglund method*

Åström-Hägglund method uses approximation description in the form of first order system with dead time. Parameter M is user specific parameter, it is optional from two values 1,4 and 2. This parameter offers the controller robustness. Higher parameter value gives faster control response and smaller improves robustness (Åström et all., 1995).

PID controller is used in the form of equation (6), with user specific constants set point weighing b, gain Z_R, integration time constant T_I and derivation time constant T_D.

$$u(t) = Z_R . \left(b.w(t) - y(t) + \frac{1}{T_I} \int_0^t e(s) ds + T_D \frac{de(t)}{dt} \right) \quad (6)$$

It is necessary to approximate the system by first order system with dead time.

$$\frac{Y(s)}{U(s)} = \frac{Z}{1+Ts} . e^{-D.s} \quad (7)$$

Three parameters (Gain Z, time constant T, dead time D) are used for controller design.

The MATLAB function fminsearch was used to identify unknown parameters from measured step response and results are shown in table 1.

Table 1 – Process approximation by first order system with dead time

Z	0,0453
T	189
D	253

Normalized parameters a and τ are defined for following calculations

$$a = Z. \frac{D}{T} \quad (8)$$

$$\tau = \frac{D}{D+T} \quad (9)$$

Controller design is empirical - it is based on simulation results with different process models. It proves that controller parameters are normalized dead time function with parameters a_0, a_1 and a_2 given in tables 2 and 3.

$$f(\tau) = a_0 . \exp(a_1 . \tau + a_2 \tau^2) \quad (10)$$

Parameters of PI and PID controller are calculated from tables 2 and 3 and shown in table 4.

Table 2 – PI controller design

	$\overline{M}$ =1,4			$\overline{M}$ =2		
	a0	a1	a2	a0	a1	a2
a.Z_R	0,29	-2,70	3,70	0,78	-4,10	5,70
T_I/D	8,90	-6,60	3,00	8,90	-6,60	3,00
T_I/T	0,79	-1,40	2,40	0,79	-1,40	2,40
b	0,81	0,73	1,90	0,44	0,78	-0,45

Table 3 – PID controller design

	$\overline{M}$ =1,4			$\overline{M}$ =2		
	a0	a1	a2	a0	a1	a2
a.Z_R	3,80	-8,40	7,30	8,40	-9,60	9,80
T_I/D	5,20	-2,50	-1,40	3,20	-1,50	-0,93
T_I/T	0,46	2,80	-2,10	0,28	3,80	-1,60
T_D/D	0,89	-0,37	-4,10	0,86	-1,90	-0,44
T_D/T	0,077	5,00	-4,80	0,076	3,40	-1,10
b	0,40	0,18	2,80	0,22	0,65	0,051

Table 4 – PI and PID controllers parameters

	PI		PID	
	$\overline{M}$ =1,4	$\overline{M}$ =2	$\overline{M}$ =1,4	$\overline{M}$ =2
Z_R	3,43	7,97	5,60	14,12
T_I	142	142	207	264
T_D	--	--	50,14	66,71
b	2,29	0,59	1,11	0,32

Two PID controllers were selected (PI controller with $\overline{M}$ =1,4, and PID controller with $\overline{M}$ =2). Control responses are shown in figures 2 and 3. Top axis shows manipulated variable behavior, set point and controlled variable are plotted in bottom part of the figure.

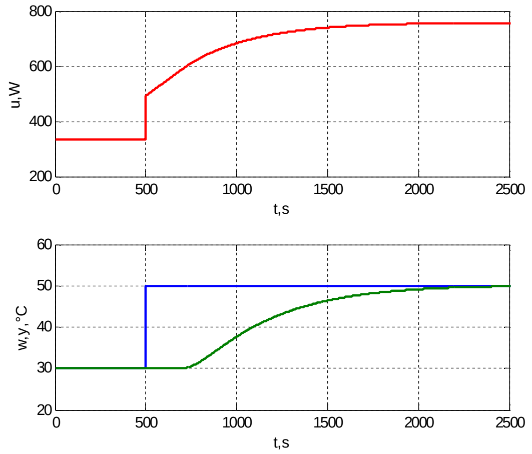

Figure 2 – PI controller $\overline{M}$ =1,4,

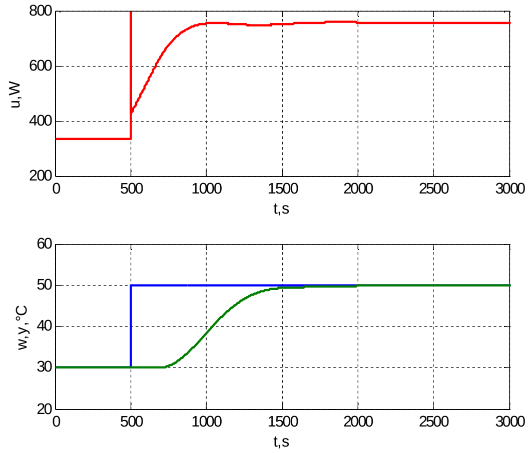

Figure 3 – PID controller $\overline{M}$ =2

4.2 *Predictive Functional Controller*

Predictive Functional Controller (PFC) belongs to family of “model predictive controllers. Process is described by model in the form of first order system with dead time (Richalet, 1993).

The calculation of manipulated variable is realized from the actual set point to decrease the control error exponentially to a fraction of its actual value at the end of control horizon. Only one manipulated variable and only one control error is used at the whole control horizon. It is possible to calculate the actual manipulated variable value using very simple algebraic equation. Manipulated variable calculation repeats in every sample time instants. The biggest PFC advantage is its design simplicity and the control law simplicity.

The approximation by first order system is the same as in Åström-Hägglund method. The results are shown in table 1 in chapter 4.1.

The principle is explained in the figure 4. Manipulated variable is plotted in the top axis, in the second there are controlled variable and set point and in the third there is model output. Actual time is in point *k* and the end of control horizon is *k+h*.

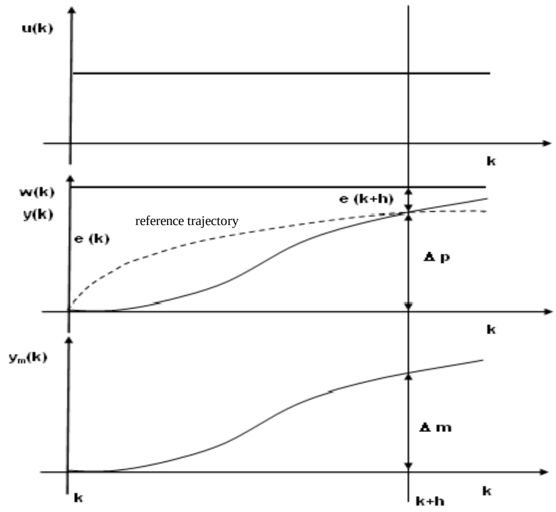

Figure 4 – PFC principle

The control law calculation assumes that model and process outputs increments are equal in time k+h.

$$\Delta p = \Delta m \tag{11}$$

If we substitute measured values from process and predicted values from model we get control action as

$$u(k) = \left[w(k) - y_P(k) - y_m(k) + y_m(k-d)\right] k_0 + y_P(k) k_1 \tag{12}$$

where constants k_0 and k_1 are calculated from (13) and (14) and λ is in equation (15).

$$k_0 = \frac{1-\lambda^h}{K.(1-a^h)} \tag{13}$$

$$k_1 = \frac{1}{K} \tag{14}$$

$$\lambda = e^{-\frac{3.\Delta t}{T_C}} \tag{15}$$

Parameters h and T_C are user specific parameters for PFC tuning. Parameter h is the control horizon and parameter T_C describes closed loop dynamics (Mareš, 2007).

PFC was used with two different sets of constants (firstly T_C=10, h=5, secondly T_C=200, h=50). Control responses are shown in figures 5 and 6. First axis describes manipulated variable behavior, set point and controlled variable are plotted in bottom part of the figure.

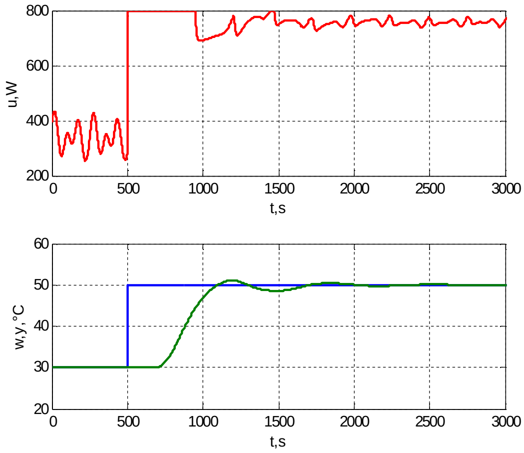

Figure 5 – PFC, T_C=10, h=5

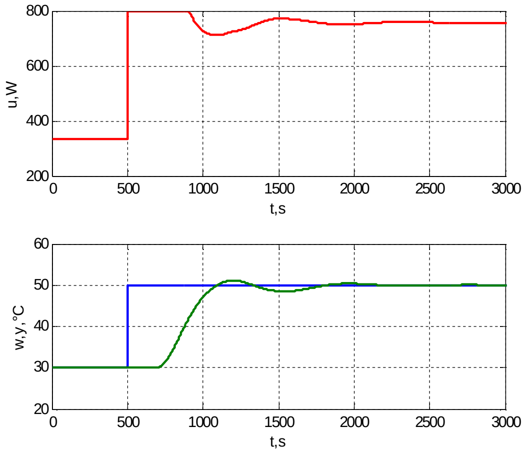

Figure 6 – PFC, T_C=200, h=50

5 CONCLUSIONS

Control of thermal process with dead time is discussed in the paper. Two different controller designs were chosen. The first one is classical PID controller with set point weighing and parameters calculated according to Åström-Hägglund method. The second one is simple predictive controller PFC.

Response of PID controller is little bit slower in comparison to PFC but without overshoot. PFC control response is faster and has a small overshoot. PFC with parameter T_C=10 has slightly oscillating manipulated variable.

If we compare the design demands and complexity, both controllers are similar. PID parameters calculation is more difficult than with PFC. On the other hand PFC concept is not so well known as classical PID. Advantage of PFC is a wide methodology open for "made to measure" solution and improvements depending on controlled process.

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