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ROBUST CONTROL OF A LABORATORY PROCESS

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Abstract: The paper presents robust control of a laboratory process with a transport delay using the industrial control system SIMATIC. The controlled process is identified in the form of a transfer function of a higher order with a transport delay at first and then the transport delay is approximated by the first order Taylor series expansion of the numerator or the denominator. Because the transport delay can vary, the controlled laboratory process is modelled in the form of a transfer function with interval parametric uncertainty. Robust PI controllers are designed for the laboratory process. The method for synthesis of robust controllers is based on plotting the stability boundary locus in the (k_p, k_i) - plane and the subsequent choice of a stabilizing PI controller using the pole-placement method so that the prescribed behavior of the closed-loop is achieved.

Keywords: PI controller, robust control, interval uncertainty, control system SIMATIC, transport delay system

1. INTRODUCTION

The field of robust control has experienced a large number of breakthroughs over last decades. The primary focal points have been robustness analysis (Kharitonov (1978)) and robustness synthesis involving structured real parametric uncertainty, see e.g. Barmish (1994). Numerous interests have grown in various problems of analysis, synthesis, and design for interval plants.

There has been done a great amount of research work on tuning of PID controllers since these types of controllers have been widely used in industrial applications. PID controller design in classical control engineering is based on a plant with fixed parameters and the latest approaches can be found e.g. in Matušů and Prokop (2008). In the real world, however, most process models are not known exactly and so, models contain uncertainties. Hence control system design for both, stability and performance robustness always requires taking uncertainties into account.

In this paper, a method for design of robust PI controllers is used, see Tan and Kaya (2003). The method is based on plotting the stability boundary locus in the (k_p, k_i)-plane and the subsequent choice of a stabilizing PI controller using the pole-placement method so that the prescribed behavior of the closed-loop is achieved.

2. SIMATIC S7-300

SIMATIC S7-300 is an industrial control system, which is used in many applications of process control. SIMATIC includes programming (STEP7) and visualization (WinCC) software, which are used for programming of programmable logic controllers (PLCs), for data accessing to users and they are simply applicable for monitoring and control of real processes.
The structure of the user's program is created by the organization block OB35, witch represents the main program that works cyclic with the sample time 100ms. The organization block OB35 includes a function block of a PID controller (FB41). Before the blocks are programmed (Kožka and Kvasnica (2001), Siemens (1996)), it is necessary to create a project, configure a network, define input and output modules and define connections between input and output modules. Visualization of the project is realized in the Graphics Designer. Visualization software WinCC gives to users a possibility to define their own visualization for controlled processes. The component of WinCC is a graphic editor. WinCC allows choice of manipulating elements, I/O fields and monitoring windows according to demands of users. WinCC processes all important data from the program STEP7 and the connection between WinCC and STEP7 is linked by tags.
For visualization of the controlled laboratory process a visualization screen has been created. All measured data and their graphic trends are displayed. User can design any objects for creating of a visualization screen individually or objects from a library can be chosen.

3. LABORATORY PROCESS

Controlled laboratory process (Fig. 1) is an electronic model of a linear 2^{nd} order system with a transport delay varying from $6s$ to $30s$ (Babirád (2006)). The process was identified by Strejc method (Fikar and Mikleš (1999)) in the form of a transfer function

$$G_s(s) = \frac{K}{(Ts+1)^2} e^{-[D_{min}, D_{max}]s} \tag{1}$$

where K is the gain, T is the time constant and D_{min}, D_{max} are the minimun and the maximum transport delays of the process. The process was identified from step responses measured in various working areas and the identified parameters are collected in Table 1.

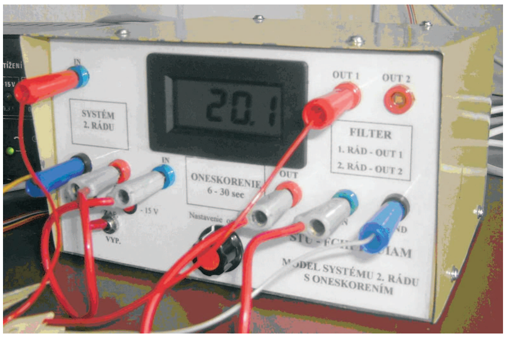

Fig. 1. Controlled laboratory process

Table 1. Identified parameters

K	$T(s)$	$D(s)$	n
0.97	10.0	12.6	2
0.97	10.1	17.1	2
0.97	10.4	22.4	2
0.97	10.7	35.4	2

4. ROBUST CONTROLLER DESIGN

4.1 *Description of an uncertain system*

Consider a system with real parametric uncertainty described by the transfer function

$$G_s(s,q) = \frac{b(s,q)}{a(s,q)} \tag{2}$$

where q is a vector of uncertain parameters and b, a are polynomials in s with coefficients which depend on a parameter q.

An uncertain polynomial

$$a(s,q) = \sum_{i=0}^{n} a_i(q) s^i \tag{3}$$

is said to have an independent uncertainty structure if each component q_i of q enters into only one coefficient a_i.
A family of polynomials

$$A = \{a(s,q) : q \in Q\} \tag{4}$$

is said to be an interval polynomial family if $a(s,q)$ has an independent uncertainty structure, each coefficient depends continuously on q and Q is a box. An interval polynomial family A arises from the uncertain polynomials described by $a(s,q)$ with uncertainty bounds $|q_i| \leq 1$ for $i = 0, \ldots, n$. When dealing with an interval family, the shorthand notation

$$a(s,q) = \sum_{i=0}^{n} [q_i^-, q_i^+] s^i \tag{5}$$

may be used with $[q_i^-, q_i^+]$ denoting the bounding interval for the i^{th} of uncertainty component of uncertainty q_i.

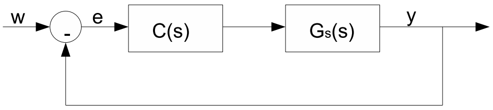

Fig. 2. Control system

4.2 *Analysis of robust stability*

In order to use the Kharitonov theorem (Kharitonov (1978)) for robust stability analysis, polynomials associated with an interval polynomial family A have to be defined at first. In the definition below the polynomials are fixed in the sense that only the bounds q_i^- and q_i^+ enter into the description but not the q_i themselves. The number of polynomials is four and they are independent on the degree of $a(s, q)$. Associated with the interval polynomial family (5) are four fixed Kharitonov polynomials (Kharitonov (1978))

$$\begin{aligned} K_1(s) &= q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + \dots \\ K_2(s) &= q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + \dots \\ K_3(s) &= q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + \dots \\ K_4(s) &= q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + \dots \end{aligned} \quad (6)$$

The interval polynomial family A with invariant degree is robustly stable if and only if its four Kharitonov polynomials (6) are stable.

4.3 *Description of PI controller synthesis*

The method of a robust PI controller synthesis (Tan and Kaya (2003)) is based on plotting the stability boundary locus in the (k_p, k_i)-plane and subsequent finding of stabilizing PI controllers. The method locates all PI controllers, which stabilize the controlled system with interval uncertainty. The stability boundary divides the parameter plane ((k_p, k_i)-plane) into stable and unstable regions. The stable ones can be determined by the choice of a test point within each region.

4.4 *Robust PI controller synthesis I*

Consider the control system in Figure 2, where $G_s(s)$ represents the controlled process with the transfer function

$$G_s(s) = \frac{-b_1 s + b_0}{a_2 s^2 + a_1 s + 1} \quad (7)$$

and $C(s)$ represents the feedback stabilizing PI controller

$$C(s) = k_p + \frac{k_i}{s} \quad (8)$$

with $k_p = Z_R$ and $k_i = \frac{Z_R}{T_I}$, where Z_R is the gain of the controller and T_I is the reset time of the controller. The closed loop characteristic equation can be written by substituting $s = j\omega$ in the form

$$\begin{aligned} &[-a_1\omega^2 + b_1 k_p \omega^2 + b_0 k_i] + \\ &\quad + j[-a_2\omega^3 + (b_0 k_p - b_1 k_i + 1)\omega] = 0 \end{aligned} \quad (9)$$

The parameters of PI controller can be easily obtained by equating the real and the imaginary parts of the characteristic equation (9) to zero, for details see Závacká et al. (2007). Equating the real and imaginary parts of (9) to zero leads following expressions for calculating of k_p, k_i in the dependence on the frequency ω

$$k_p = \frac{\omega^2(a_2 b_0 + a_1 b_1) - b_0}{b_0^2 + b_1^2 \omega^2} \quad (10a)$$

$$k_i = \frac{\omega^2(a_1 - k_p b_1)}{b_0} \quad (10b)$$

4.5 *Robust PI controller synthesis II*

Consider further the control system in Figure 2 where $G_s(s)$ represents the controlled process with the transfer function

$$G_s(s) = \frac{b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \quad (11)$$

and $C(s)$ is the feedback stabilizing PI controller (8). The closed loop characteristic equation after the substitution $s = j\omega$ is

$$\begin{aligned} &[a_3\omega^4 - a_2\omega^2 + b_0 k_i] + \\ &\quad + j[-a_2\omega^3 + (b_0 k_p + a_0)\omega] = 0 \end{aligned} \quad (12)$$

Equating the real and imaginary parts of the characteristic equation to zero gives following expressions for calculating of k_p, k_i in the dependence on the frequency ω

$$k_p = \frac{a_2\omega^2 + a_0}{b_0} \quad (13a)$$

$$k_i = \frac{-a_3\omega^4 + a_1\omega^2}{b_0} \quad (13b)$$

5. POLE-PLACEMENT METHOD

For the control system in Figure 2, where $G_s(s)$ represents the controlled system of the 2^{nd} or the 3^{rd} order and $C(s)$ represents the PI controller (8), the closed loop characteristic equation can be

$$(s + c_1)(s^2 + 2\xi\omega_0 s + \omega_0^2) = 0 \quad (14a)$$

$$(s + c_1)^2(s^2 + 2\xi\omega_0 s + \omega_0^2) = 0 \quad (14b)$$

where ξ is the relative damping, ω_0 is the natural undamped frequency and $-c_1$ is the pole of the closed-loop system (Bakošová and Fikar (2008)). The closed loop characteristic equation for the considered controlled systems (7) or (11) and the PI controller has the form

$$s^3 + \frac{a_1 - b_1 k_p}{a_2} s^2 + \\ + \frac{a_0 + b_0 k_p - b_1 k_i}{a_2} s + \frac{b_0 k_i}{a_2} = 0 \quad (15a)$$

or

$$s^4 + \frac{a_2}{a_3} s^3 + \frac{a_1}{a_3} s^3 + \\ + \frac{a_0 + b_0 k_p}{a_3} s + \frac{b_0 k_i}{a_3} = 0 \quad (15b)$$

After the suitable choice of ξ, ω_0, c_1 and comparison of coefficients in (14a) and (15a), the parameters of the PI controller can be computed as follows

$$k_p = \frac{a_2(c_1 + 2\xi\omega_0) - a_1}{-b_1} \quad (16a)$$

$$k_i = \frac{c_1 a_2 \omega_0^2}{b_0} \quad (16b)$$

Simularly, after the suitable choice of ξ, ω_0, c_1 and comparison of coefficients in (14b) and (15b), the parameters of the PI controller can be computed as follows

$$k_p = \frac{a_3(2\xi\omega_0 c_1^2 + 2c_1\omega_0^2) - a_0}{b_0} \quad (17a)$$

$$k_i = \frac{c_1^2 a_2 \omega_0^2}{b_0} \quad (17b)$$

The pole-placement method was used for the nominal model of the controlled process with following values of identified parameters: the gain $K = 0.97$, the time constant $T = 10.3s$ and the transport delay $D_{nom} = 24.0s$.

6. RESULTS

6.1 *Application of robust controller synthesis I and pole-placement method*

The identified transfer function (1) of the laboratory process was modified by approximation of the transport delay. The term representing the transport delay in (1) was aproximated by its 1st order Taylor series expansion of the numerator. So, the modified transfer function has the form (7) where

$$\begin{aligned} a_2 &= T^2 \\ a_1 &= 2T \\ a_0 &= 1 \\ b_1 &= [KD^-, KD^+] \\ b_0 &= K. \end{aligned} \quad (18)$$

It can be stated according to (18) that the controlled process is a system with parametric interval uncertainty.

Table 2. The parameters of PI controllers I

	Z_R	T_I
$C1$	0.10	7.0
$C2$	0.20	29.1

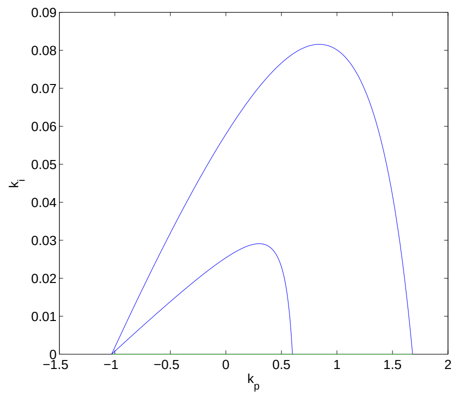

Fig. 3. Stability region for (18)

The parameters of robust PI controllers were found by the method described in the part 4.4 and 5.

In the stability region (Fig. 3) were found parameters of PI controllers (16) for following choice of characterictic equation parameters: $\xi = 0.77$, $c_1 \in [0.42 : 0]$ and $\omega_0 \in [0 : 0.10]$. Found k_p and k_i lie on the magenta line in Figure 4. From designed k_p, k_i parameters, two PI controllers were chosen: $C1$ with $\xi = 0.77$, $c_1 = 0.12$ and $\omega_0 = 0.03$, and $C2$ with $\xi = 0.77$, $c_1 = 0.11$ and $\omega_0 = 0.02$ (Tab. 2) (green stars in Figure 4).

The robust stability of the designed feedback control loop was also tested and the Kharitonov theorem was used. The characteristic equation of the feedback control loop is

$$1 + C(s)G_s(s) = 0 \quad (19)$$

where parameters of $G_s(s)$ are given in (18) and parameters of $C(s)$ are given in Table (2). Four fixed Kharitonov polynomials for the characteristic equation were created according to (6) and their stability was tested. The command `kharit` from the Polynomial Toolbox was used and the result of this test is, that the polynomial on the left side of (19) is robustly stable. It means that the feedback control loop with designed controllers is robustly stable.

6.2 *Application of robust controller synthesis and pole-placement method II*

The identified transfer function (1) was modified by approximation of the transport delay. The term representing the transport delay in (1) was substituted by its 1st order Taylor series expansion of

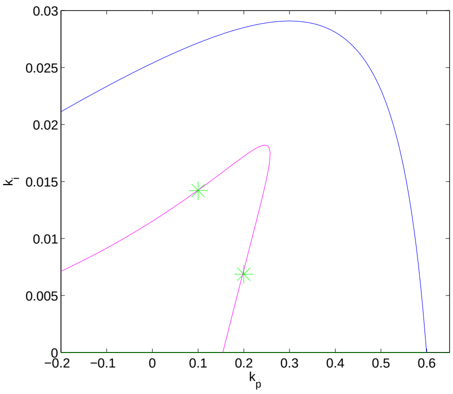

Fig. 4. The position of controllers in the stability region for (18)

Table 3. The parameters of PI controller II

	Z_R	T_I
$C3$	0.50	34.3

the denominator. So the transfer function has the form (11) where

$$\begin{aligned} a_3 &= [T^2D^-, T^2D^+] \\ a_2 &= [T^2 + 2TD^-, T^2 + 2TD^+] \\ a_1 &= [2T + D^-, 2T + D^+] \\ a_0 &= 1 \\ b_0 &= K. \end{aligned} \tag{20}$$

According to (20), it can be stated, that the controlled process is a system with parametric interval uncertainty.
The parameters of robust PI controllers were found by the method described in the part 4.5 and 5.

In the stability region (Fig. 5), parameters of PI controllers (17) were found for $\xi = 1.1$, $c_1 \in [0.12 : 0.06]$ and $\omega_0 \in [0 : 0.05]$. Found k_p and k_i lie on the magenta line in Figure 6. From designed k_p, k_i parameters was chosen the PI controller $C3$ with $\xi = 1.1$, $c_1 = 0.09$ and $\omega_0 = 0.03$ (Tab. 3)(green star in Figure 6).

The robust stability of the designed feedback control loop was also tested using the Kharitonov theorem by the way as it was described in the previous section. The result of the test confirmed that the polynomial (19) is robustly stable. It means the feedback control loop with the designed controller $C3$ is robustly stable.

6.3 *Control of laboratory process*

The laboratory process was controlled using the robust PI controllers (Tab. 2) and (Tab. 3). These controllers were implemented via the control system SIMATIC.

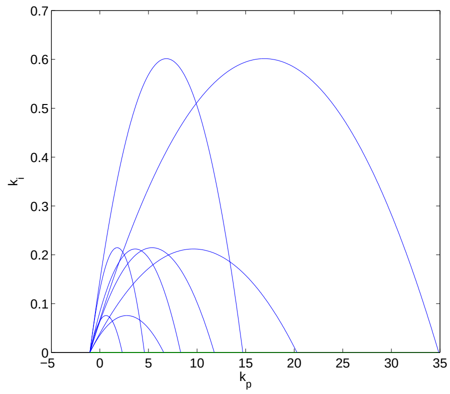

Fig. 5. Stability region for (20)

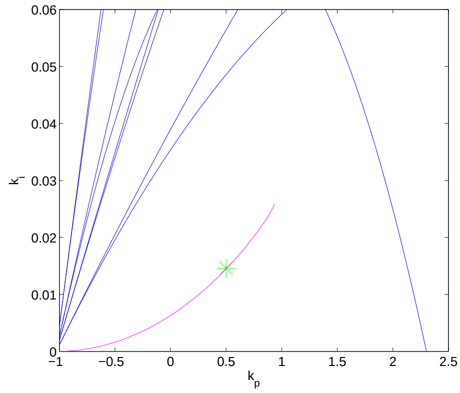

Fig. 6. The position of controllers in stability region for (20)

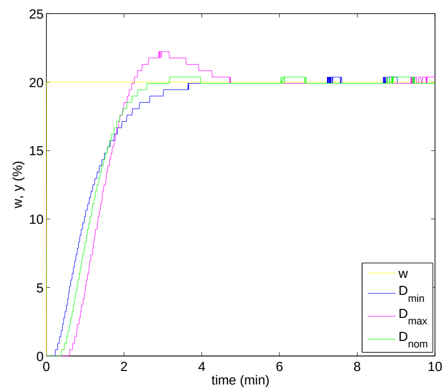

Fig. 7. Time responses for controller $C1$

Time responses of the closed loop with the controlled process with different values of the transport delay and the PI controller $C1$ are shown in Figure 7, where w is the setpoint and D_{min}, D_{max}, D_{nom} are minimal, maximal and nominal transport delays.

Time responses of the closed loop with the controlled process with different values of transport delay and the PI controller C_2 are shown in Figure 8, where w is the setpoint and D_{min}, D_{max},

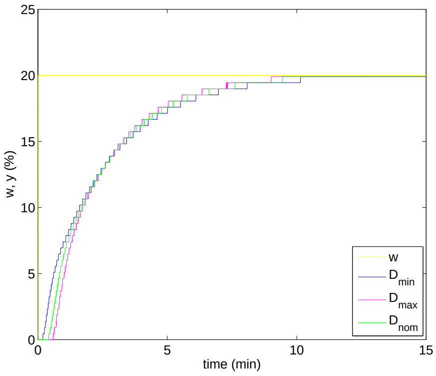

Fig. 8. Time responses for controller $C2$

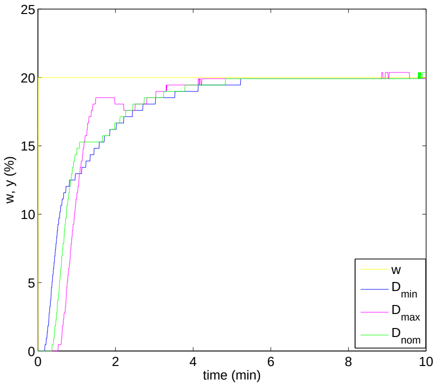

Fig. 9. Time responses for controller $C3$

D_{nom} are minimal, maximal and nominal transport delays.

Time responses of the closed loop with the controlled process with different values of transport delay and the PI controller C_3 are shown in Figure 9 where w is the setpoint and D_{min}, D_{max}, D_{nom} are minimal, maximal and nominal transport delays.

CONCLUSIONS

The electronic model was identified as the process with interval uncertainty in the transport delay. For this process, robust PI controllers were designed by combination of two methods: the method based on the stability boundary locus in the (k_p, k_i)-plane and the pole-placement method. Adding the pole-placement method to the robust PI controller design offers the possibility to assure the prescribed behavior of the closed loop given by the choice of ξ, c_1 and ω_0. Designed controllers were implemeted for control of the laboratory process using the control system SIMATIC. Obtained experimental results confirm that the designed robust PI controllers are suitable for control of real processes with varying transport delay.

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