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ADAPTIVE SLIDING-MODE CONTROL OF NONLINEAR SYSTEMS USING NEURAL NETWORK APPROACH

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Abstract: This paper is concerned with the adaptive sliding-mode control of nonlinear dynamic systems with model uncertainties. The proposed control method combines the advantages of sliding-mode control and backstepping methodology, such that the requirement of the restrictive matching condition is removed, which seriously clams the application of sliding-mode control. In the control scheme, networks of Gaussian radial basis functions with variable weights are used to compensate the model uncertainties. The adaptive law developed using the Lyapunov synthesis approach guarantees the stability of the control scheme. The performance is illustrated by experimental studies with a flexible-joint manipulator.

Keywords: Adaptive control, Sliding-mode control, Backstepping, Gaussian radial-basis-function networks, Nonlinear systems

1. INTRODUCTION

The robustness in the face of model uncertainties in a control system with a sliding-mode controller (SMC) is due to the high-frequency switching term of the control, which in practice equals to a continuous high-gain control as shown by Utkin et al. (1999). The switching gain has to be higher than the known norm of the uncertainties. When the uncertainties grow beyond this bound, the switching controller is no longer capable of maintaining the sliding mode, and the system loses robustness to uncertainties and disturbances. A more conservative estimation of the uncertainties may help to maintain the stability but leads to a higher control gain and more control effort. Furthermore, this may also lead to problems with parasitic dynamics of the system as shown by Young and Kokotovic (1982). It is then necessary to extend the standard SMC to an adaptive one following Slotine and Coetsee (1986). However, classical parameter estimation methods and adaptive control schemes require that the system model be linearly parameterised and that the nonlinearities are exactly known. In general, this is seldom the case. Another drawback of SMC is the requirement of the matching condition. As for deterministic robust control, SMC requires that the uncertainties and disturbances can be lumped into the input channel, so that they can be efficiently compensated by the control input according to Drazenović (1969) and Gutman (1979). However, the matching condition is a very strong assumption of the system structure. This seriously clams the application of the SMC method.

The backstepping method of Kanellakopoulos et al. (1991) is a breakthrough for adaptive nonlinear control. This provides a systematic procedure to construct a robust control Lyapunov function. It is natural to integrate the backstepping algorithm into the design of a SMC. This removes the requirement of the matching condition, and the powerful SMC technique can provide robustness of the adaptive system. Two basic ideas are known from literature: 1) A sliding surface as a linear combination of the control errors is constructed at the final step of the backstepping. A SMC term ensures the convergence of the system states to the sliding surface, so that the control errors also converge, see Rios-Bolivar et al. (1997). 2) Sliding surfaces are constructed in each step of the backstepping, so that the convergence of the system states is progressively approached as shown by Huang and Chen (2004).

In this paper, backstepping design is utilised to remove the problem of mismatched uncertainties for a class of dynamic systems in nonlinear parametric-pure-feedback form (NPPF). A single sliding surface is constructed in the last step of backstepping. Compared with the method proposed in Rios-Bolivar et al. (1997), the stability analysis and the control law are considerably simplified. In the proposed control scheme, networks of Gaussian radial basis functions (GRBF) according to Ma (2005) are used to approximate the model uncertainties. The reason of choosing these networks is that their outputs are linear combinations of the neurons outputs, such that the stability of the overall system can be easier achieved. The sliding-mode control term needs only to deal with the approximation errors of the networks, such that large, conservative switching gains can be avoided. The updating law of the networks is obtained by the Lyapunov design.

The remainder of this paper is organised as follows. In section 2 , the adaptive backstepping SMC scheme is presented. The stability of the overall system is analysed. The performance of the presented control scheme is demonstrated in section 3 by experimental studies of the tracking control of a flexible-joint robot manipulator. A brief conclusion is given in section 4.

2. THE ADAPTIVE BACKSTEPPING SLIDING-MODE CONTROL SCHEME

2.1 *Problem statement*

Consider systems in the nonlinear parametric-pure-feedback form (NPPF)

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(x_1, \ldots, x_{i+1}, \boldsymbol{\theta}), \qquad 1 \le i \le n-1, \\ \dot{x}_n &= b(\boldsymbol{x})u + f_n(\boldsymbol{x}), \end{aligned} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ is the vector of the system states. $f_1, f_2, \ldots, f_n$ are unknown yet smooth scalar nonlinear functions. $\boldsymbol{\theta}$ presents the unknown parameters in the model. The control objective is to track a desired trajectory x_{d} with x_1, assuming that all system states $x_1, x_2, \ldots, x_n$ and the derivatives of the desired trajectory $\dot{x}_{\mathrm{d}}, \ddot{x}_{\mathrm{d}}, \ldots, x_{\mathrm{d}}^{(n)}$ are available for control design.

Using GRBF networks to approximate the unknown functions, Eq.(1) can be rewritten as

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \boldsymbol{W}_i \boldsymbol{\phi}_i(x_1, \ldots,\ x_{i+1}), 1 \le i \le n-1, \\ \dot{x}_n &= b(\boldsymbol{x})u + \boldsymbol{W}_n \boldsymbol{\phi}_n(x_1, \ldots,\ x_n). \end{aligned} \tag{2}$$

For the the i-th network with m_i basis functions, $\boldsymbol{W}_i = [w_{i,1}, \ldots, w_{i,m_i}]$ is a row vector of the output weights, $\boldsymbol{\phi}_i = [\phi_{i,1}, \ldots, \phi_{i,m_i}]^{\mathrm{T}}$ is a column vector of the output of the basis functions with

$$\phi_{i,j} = e^{-\frac{1}{2\sigma_{i,j}^2} \sum_{k=1}^{k_{\max}} (x_k - \xi_{i,j}^k)^2}, 1 \le j \le m_i, \tag{3}$$

where $k_{\max} = i + 1$ for $i < n$ and $k_{\max} = n$ for $i = n$. $\sigma_{i,j}$ is the width and $\xi_{i,j}^k$ is the center of the (i,j)-th basis function with respect to the k-th input x_k.

Using a network arranged on a regular lattice following Sanner and Slotine (1992), there exists an optimal output-weight vector $\boldsymbol{W}_i^*$ and a positive scalar ε_i^0 such that

$$f_i = \boldsymbol{W}_i^* \boldsymbol{\phi}_i(x_1, \ldots,\ x_{k_{\max}}) + \epsilon_i, \qquad |\epsilon_i| \le \epsilon_i^0, \tag{4}$$

where ϵ_i $i = 1, \ldots, n$ is the approximation error of the i-th GRBF network. Define $\hat{\boldsymbol{W}}_i$ as the estimation of $\boldsymbol{W}_i^*$, and the estimation error $\tilde{\boldsymbol{W}}_i = \boldsymbol{W}_i^* - \hat{\boldsymbol{W}}_i$, Eq.(4) becomes

$$f_i = \hat{\boldsymbol{W}}_i \boldsymbol{\phi}_i + \tilde{\boldsymbol{W}}_i \boldsymbol{\phi}_i + \epsilon_i, \qquad |\epsilon_i| \le \epsilon_i^0. \tag{5}$$

Let $\boldsymbol{W} = [\boldsymbol{W}_1,\ \ldots,\ \boldsymbol{W}_n]$,

and $\boldsymbol{\Phi}_i = [\mathbf{0}^{\mathrm{T}}_{(1\times\sum_{j=1}^{i-1} m_j)} \vdots\ \boldsymbol{\phi}_i^{\mathrm{T}} \vdots\ \mathbf{0}^{\mathrm{T}}_{(1\times\sum_{j=i+1}^{n} m_j)}]^{\mathrm{T}}$,
Eq.(2) is rewritten as

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \boldsymbol{W}\boldsymbol{\Phi}_i(x_1, \ldots,\ x_{i+1}) + \epsilon_i, 1 \le i \le n-1, \\ \dot{x}_n &= b(\boldsymbol{x})u + \boldsymbol{W}\boldsymbol{\Phi}_n(x_1, \ldots,\ x_n) + \epsilon_n. \end{aligned} \tag{6}$$

2.2 *Backstepping design procedure*

Step 1: Define the tracking error

$$z_1 = x_1 - x_{\mathrm{d}} \tag{7}$$

and positive constants $c_1, \ldots, c_n$, $g_1, \ldots, g_n$. The first derivative of the control error is

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{x}_{\mathrm{d}} \\ &= x_2 + f_1(x_1, x_2, \boldsymbol{\theta}) - \dot{x}_{\mathrm{d}}. \end{aligned} \tag{8}$$

Treating x_2 as a control signal for Eq.(8), the control law $x_{2\mathrm{d}}$ for x_2 which stabilises z_1 would be

$$x_{2\mathrm{d}} = -c_1 z_1 - f_1 + \dot{x}_{\mathrm{d}}. \tag{9}$$

Since f_1 is unknown, a GRBF network is used for approximation such that the actual value of $x_{2\mathrm{d}}$ is

$$x_{2\mathrm{d}} = -c_1 z_1 - \hat{\boldsymbol{W}}\boldsymbol{\Phi}_1(x_1, x_2) + \dot{x}_{\mathrm{d}}. \tag{10}$$

Define z_2 as the difference between x_2 and $x_{2\mathrm{d}}$ as

$$z_2 = x_2 + c_1 z_1 + \hat{\boldsymbol{W}}\boldsymbol{\Phi}_1 - \dot{x}_{\mathrm{d}}. \tag{11}$$

Substituting Eq.(11) into Eq.(8), $\dot{z}_1$ becomes

$$\dot{z}_1 = -c_1 z_1 + z_2 + \tilde{\boldsymbol{W}}\boldsymbol{\Phi}_1 + \epsilon_1. \quad (12)$$

Furthermore, let

$$\alpha_1(x_1, x_2, x_\mathrm{d}, \hat{\boldsymbol{W}}) = -c_1 z_1 - \hat{\boldsymbol{W}}\boldsymbol{\Phi}_1, \quad (13)$$

z_2 is written as

$$z_2 = x_2 - \alpha_1 - \dot{x}_\mathrm{d}. \quad (14)$$

Step 2: The derivative of z_2 is

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 - \ddot{x}_\mathrm{d} \\ &= x_3 + \boldsymbol{W}\boldsymbol{\Phi}_2 + \epsilon_2 - \frac{\partial\alpha_1}{\partial x_1}(x_2 + \boldsymbol{W}\boldsymbol{\Phi}_1 + \epsilon_1) \\ &\quad - \frac{\partial\alpha_1}{\partial x_2}(x_3 + \boldsymbol{W}\boldsymbol{\Phi}_2 + \epsilon_2) \\ &\quad - \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{W}}_1}\dot{\hat{\boldsymbol{W}}}_1^\mathrm{T} - \frac{\partial\alpha_1}{\partial x_\mathrm{d}}\dot{x}_\mathrm{d} - \ddot{x}_\mathrm{d}. \end{aligned} \quad (15)$$

By ignoring $\frac{\partial\alpha_1}{\partial x_1}\epsilon_1$ and $\frac{\partial\alpha_1}{\partial x_2}\epsilon_2$, Eq.(15) becomes

$$\begin{aligned}\dot{z}_2 &= x_3 + \epsilon_2 - \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial x_i}x_{i+1} - \frac{\partial\alpha_1}{\partial x_\mathrm{d}}\dot{x}_\mathrm{d} - \ddot{x}_\mathrm{d} \\ &\quad - \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{W}}_1}\dot{\hat{\boldsymbol{W}}}_1^\mathrm{T} + \boldsymbol{W}(\boldsymbol{\Phi}_2 - \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial x_i}\boldsymbol{\Phi}_i). \end{aligned} \quad (16)$$

Let

$$\begin{aligned}\alpha_2 &= -c_2 z_2 - z_1 + \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial x_i}x_{i+1} + \frac{\partial\alpha_1}{\partial x_\mathrm{d}}\dot{x}_\mathrm{d} \\ &\quad + \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{W}}_1}\dot{\hat{\boldsymbol{W}}}_1^\mathrm{T} - \hat{\boldsymbol{W}}(\boldsymbol{\Phi}_2 - \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial x_i}\boldsymbol{\Phi}_i), \end{aligned} \quad (17)$$

the control law for x_3 to stabilise $\dot{z}_2$ would be

$$x_{3\mathrm{d}} = \alpha_2 + \ddot{x}_\mathrm{d}, \quad (18)$$

and the difference between x_3 and its desired value $x_{3\mathrm{d}}$ is

$$\begin{aligned}z_3 &= x_3 + c_2 z_2 + z_1 - \ddot{x}_\mathrm{d} - \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial x_i}x_{i+1} - \frac{\partial\alpha_1}{\partial x_\mathrm{d}}\dot{x}_\mathrm{d} \\ &\quad - \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{W}}_1}\dot{\hat{\boldsymbol{W}}}_1^\mathrm{T} + \hat{\boldsymbol{W}}(\boldsymbol{\Phi}_2 - \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial x_i}\boldsymbol{\Phi}_i). \end{aligned} \quad (19)$$

Substituting Eq.(19) into Eq.(15), $\dot{z}_2$ becomes

$$\dot{z}_2 = -c_2 z_2 + z_3 - z_1 + \epsilon_2 + \tilde{\boldsymbol{W}}(\boldsymbol{\Phi}_2 - \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial x_i}\boldsymbol{\Phi}_i). \quad (20)$$

Step i ($1 \le i \le n-1$): The derivative of z_i is

$$\begin{aligned}\dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} - x_\mathrm{d}^{(i)} \\ &= x_{i+1} + \epsilon_i - x_\mathrm{d}^{(i)} - \frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{W}}_{i-1}}\dot{\hat{\boldsymbol{W}}}_{i-1}^\mathrm{T} \\ &\quad + \boldsymbol{W}(\boldsymbol{\Phi}_i - \sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{\Phi}_j) \\ &\quad - \sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1} - \frac{\partial\alpha_{i-1}}{\partial x_\mathrm{d}^{(i-2)}}x_\mathrm{d}^{(i-1)}. \end{aligned} \quad (21)$$

Let

$$\begin{aligned}\alpha_i &= -c_i z_i - z_{i-1} - \hat{\boldsymbol{W}}(\boldsymbol{\Phi}_i - \sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{\Phi}_j) \\ &\quad + \sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1} + \frac{\partial\alpha_{i-1}}{\partial x_\mathrm{d}^{(i-2)}}x_\mathrm{d}^{(i-1)} \\ &\quad + \frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{W}}_{i-1}}\dot{\hat{\boldsymbol{W}}}_{i-1}^\mathrm{T}, \end{aligned} \quad (22)$$

and define z_{i+1} as

$$z_{i+1} = x_{i+1} - \alpha_i - x_d^{(i)}, \quad (23)$$

note that $z_0 = 0$, then for $i < n$, Eq.(21) becomes

$$\dot{z}_i = -c_i z_i + z_{i+1} - z_{i-1} + \epsilon_i + \tilde{\boldsymbol{W}}\boldsymbol{\beta}_i, \quad (24)$$

with

$$\boldsymbol{\beta}_i = \boldsymbol{\Phi}_i - \sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{\Phi}_j. \quad (25)$$

Step n: Let z_n be the difference between x_n and its desired value $x_{n\mathrm{d}}$, its derivative is

$$\begin{aligned}\dot{z}_n &= b(\boldsymbol{x})u + \epsilon_n - x_\mathrm{d}^{(n)} - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}x_{i+1} \\ &\quad - \frac{\partial\alpha_{n-1}}{\partial x_\mathrm{d}^{(n-2)}}x_\mathrm{d}^{(n-1)} + \boldsymbol{W}(\boldsymbol{\Phi}_n - \sum_{i=1}^{n}\frac{\partial\alpha_{n-1}}{\partial x_i}\boldsymbol{\Phi}_i) \\ &\quad + \frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol{W}}_{n-1}}\dot{\hat{\boldsymbol{W}}}_{n-1}^\mathrm{T} + \frac{\partial\alpha_{n-1}}{\partial x_n}bu. \end{aligned} \quad (26)$$

Define the sliding surface as

$$s = \sum_{i=1}^{n} g_i z_i, \qquad g_i > 0, g_n = 1, \quad (27)$$

let

$$u^* = \sum_{i=1}^{n-1} g_i(-c_i z_i + z_{i+1} - z_{i-1}), \quad (28)$$

and

$$u_\mathrm{smc} = -\rho\frac{s}{|s|}, \qquad \rho > \sum_{i=1}^{n} g_i\epsilon_i^0, \quad (29)$$

the main result of this paper is presented in the following theorem:

Theorem 1. For the dynamic system in the NPPF form according to Eq.(1), the tracking error $x_1 - x_\mathrm{d}$ converges to zero asymptotically with the control

$$\begin{aligned}u &= \frac{1}{\tilde{b}(\boldsymbol{x})}[x_\mathrm{d}^{(n)} + \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}x_{i+1} + \frac{\partial\alpha_{n-1}}{\partial x_\mathrm{d}^{(n-2)}}x_\mathrm{d}^{(n-1)} \\ &\quad - \hat{\boldsymbol{W}}(\boldsymbol{\Phi}_n - \sum_{i=1}^{n}\frac{\partial\alpha_{n-1}}{\partial x_i}\boldsymbol{\Phi}_i) \\ &\quad - \frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol{W}}_{n-1}}\dot{\hat{\boldsymbol{W}}}_{n-1}^\mathrm{T} - u^* + u_\mathrm{smc}], \end{aligned} \quad (30)$$

with $\tilde{b}(\boldsymbol{x}) = \frac{1}{b(\boldsymbol{x})(1+\frac{\partial\alpha_{n-1}}{\partial x_n})}$.

The stability analysis is presented as follows: Substitute Eq.(30) into Eq.(26), $\dot{z}_n$ becomes

$$\dot{z}_n = \epsilon_n + \tilde{\boldsymbol{W}}(\boldsymbol{\Phi}_n - \sum_{i=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_i}\boldsymbol{\Phi}_i) - u^* + u_{\text{smc}}. \tag{31}$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2}s^2 + \frac{1}{2}\gamma^{-1}\tilde{\boldsymbol{W}}\tilde{\boldsymbol{W}}^{\text{T}}, \qquad \gamma > 0, \tag{32}$$

the derivative of this function is

$$\dot{V} = s(\sum_{i=1}^{n} g_i \dot{z}_i) + \gamma^{-1}\tilde{\boldsymbol{W}}\dot{\tilde{\boldsymbol{W}}}^{\text{T}}. \tag{33}$$

Substitute Eq.(24) and Eq.(31) into Eq.(33), it follows that

$$\dot{V} = s[\sum_{i=1}^{n} g_i(\epsilon_i + \tilde{\boldsymbol{W}}\boldsymbol{\beta}_i) + u_{\text{smc}}] + \gamma^{-1}\tilde{\boldsymbol{W}}\dot{\tilde{\boldsymbol{W}}}^{\text{T}}. \tag{34}$$

Let the updating law of $\hat{\boldsymbol{W}}$ be

$$\dot{\hat{\boldsymbol{W}}}^{\text{T}} = -\dot{\tilde{\boldsymbol{W}}}^{\text{T}} = \gamma s \sum_{i=1}^{n} g_i \boldsymbol{\beta}_i, \tag{35}$$

one has

$$\begin{aligned}\dot{V} &= s[\sum_{i=1}^{n} g_i\epsilon_i + u_{\text{smc}}] = s\sum_{i=1}^{n} g_i\epsilon_i - \rho|s| \\ &\le |s|(\sum_{i=1}^{n} g_i\epsilon_i^0 - \rho).\end{aligned} \tag{36}$$

Recall that $\rho > \sum_{i=1}^{n} g_i\epsilon_i^0$, then it is true that $\dot{V} < 0$ such that s and $\tilde{W}$ converge to zero asymptotically. According to Eq.(27), z_i also converges to zero for $1 \le i \le n$. Specially, it is true that $x_1 - x_{\text{d}} \to 0$ as $t \to 0$, viz. the tracking error converges to zero asymptotically.

The input function $b(\boldsymbol{x})$ is assumed to be known. For real control objectives, $b(\boldsymbol{x})$ is nonzero and bounded in order to fulfill the controllability of the system. Furthermore, α_{n-1} depends on the design parameters, such that $\tilde{b}(\boldsymbol{x})$ can be selected as nonzero and Eq.(30) is appropriate. If $b(\boldsymbol{x})$ is unknown, it can be also approximated by an additional network. In this case, some projection methods are needed in order to avoid possible singularities in $b(\boldsymbol{x})$.

Regarding the adaptive backstepping SMC in Rios-Bolivar et al. (1997) and Koshkouei et al. (2002), the Lyapunov function is defined as $V = \frac{1}{2}\sum_{i=1}^{n-1} z_i^2 + \frac{1}{2}s^2 + \frac{1}{2}\gamma^{-1}\tilde{\boldsymbol{\theta}}\tilde{\boldsymbol{\theta}}^{\text{T}}$, where $\boldsymbol{\theta}$ is the vector of unknown parameters. With the control scheme proposed here, the redundant term of z_i already included in s is omitted, such that both, the stability analysis and the control law, are considerably simplified.

3. EXPERIMENTAL RESULTS

For investigation of the proposed adaptive sliding-mode control scheme the single-link flexible-joint robot arm in Figure 1 of Quanser is used. By ignoring the viscous friction, the dynamic model of this flexible joint module moving vertically is

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{mgh}{J_{\text{Arm}}}\sin(x_1) - \frac{1}{J_{\text{Arm}}}k(x_1 - x_3), \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \frac{k}{J_{\text{eq}}}(x_1 - x_3) + \frac{1}{J_{\text{eq}}}u,\end{aligned} \tag{37}$$

where x_1 and x_3 are the link and joint angular position, respectively, $x_1 - x_3$ represent the joint deflection caused by the springs, k is the joint stiffness, J_{Arm} is the inertia of the rigid beam and J_{eq} is the inertia of the motor with the gearbox and the frame, h is the height of the center of gravity of the link with respect to the rest point, g is the gravity constant, u is the torque produced by the motor.

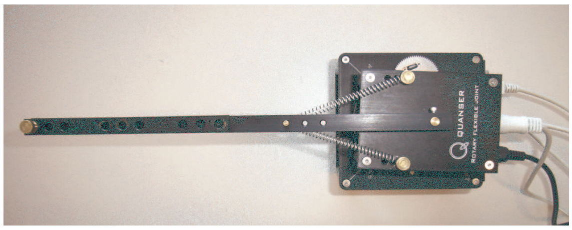

Fig. 1. The flexible-joint robot arm

In order to simplify the design procedure, the singular-perturbation method of Spong (1990) is used to transfer the dynamics into a fast part and a rigid part such that the dynamics of the rigid part can be represented with $J = J_{\text{Arm}} + J_{\text{eq}}$ as

$$J\ddot{x}_1 + mgh\sin(x_1) = u_s, \tag{38}$$

where u_s is the slow control for the equivalent rigid manipulator. With respect to the fast dynamics, if the system dynamics are exactly known, it is possible to design a state-feedback controller in order to place the poles of the closed-loop system, and thus damp out the joint oscillations in the fast time scale, otherwise a simple PD control term would be able to stabilise the joint deflection.

The parameters of this system are: $J_{\text{Arm}} = 0.0019\text{kg.m}^2$, $J_{\text{eq}} = 0.0026\text{kg.m}^2$, $k = 3.03\text{Nm/rad}$, $m = 0.064\text{kg}$, $h = 0.15\text{m}$, motor efficiency $\eta_{\text{m}} = 0.69$, gearbox efficiency $\eta_{\text{g}} = 0.9$, back-emf constant $K_{\text{m}} = 0.00767$, motor-ratio torque constant $K_{\text{t}} = 0.00767$, Gear $K_{\text{g}} = 70$, and motor resistance $R_{\text{m}} = 2.6\Omega$. The torque produced by the DC motor is given as

$$u = k_1 v - k_2\dot{x}_3, \tag{39}$$

with $k_1 = \frac{\eta_m \eta_g K_t K_g}{R_m} = 0.1282$, $k_2 = \frac{\eta_m \eta_g K_m K_t K_g^2}{R_m} = 0.0689$, and v is the armature voltage of the DC motor, which represents the real control effort.

For the control design, Eq.(38) becomes

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f(x_1, x_2) + bv \end{aligned} \tag{40}$$

with $f(x_1, x_2) = -\frac{1}{J}(mgh\sin(x_1) + k_2\dot{x}_1)$ and $b = \frac{k_1}{J}$. Define $z_1 = x_1 - x_d$ and $z_2 = x_2 + c_1 z_1 - \dot{x}_d$, the control of the rigid part of the dynamics can also be represented by the sum of three parts: the output of the GRBF network v_{nn}, the backstepping term v_{bst} and the switching control term v_{smc}:

$$v_s = \frac{1}{b}(v_{bst} + v_{smc} + v_{nn}) \tag{41}$$

with

$$v_{bst} = \ddot{x}_d - c_1\dot{z}_1 - g_1(z_2 + c_1 z_1),$$
$$v_{smc} = -\rho\frac{s}{|s|}, \qquad \rho > 0$$

and

$$v_{nn} = \hat{\boldsymbol{W}}\boldsymbol{\Phi},$$

where $\hat{\boldsymbol{W}}$ is the output weight matrix of the GRBF network and $\boldsymbol{\Phi}$ is the output vector of the hidden neurons.

A GRBF network arranged on a 2-D regular grid with 9 hidden nodes is applied. The grid points are [0, 0.6 , 1.2] on the x_1-axis and $[-1.5, 0, 1.5]$ on the $\dot{x}_1$-axis. The widths of the Gaussian functions are chosen as $\sigma_1 = 0.5\pi$ and $\sigma_2 = \pi$, respectively. The initial value of the output weights are set to zero. The adaptation gain of the network output weights is $\gamma = 5$. Other parameters are $c_1 = 3$, $g_1 = 10$, $\rho = 0.1$.

The frequency of the armature voltage v cannot be higher than 50Hz in order to protect the gearbox. This gives a higher bound of the control bandwidth. Consider Eq.(34), one has

$$\begin{aligned} \dot{s} &= \sum_{i=1}^{n} g_i(\epsilon_i + \tilde{\boldsymbol{W}}\boldsymbol{\beta}_i) + u_{smc} \\ &= -\rho\frac{s}{|s|} + \sum_{i=1}^{n} g_i(\epsilon_i + \tilde{\boldsymbol{W}}\boldsymbol{\beta}_i). \end{aligned} \tag{42}$$

In order to avoid high-frequency control, the sliding control term is linearised in a boundary layer as $u_{smc} = -\rho\frac{s}{\psi}$ as $|s| < \psi$, where ψ is thickness of the boundary, such that Eq.(42) becomes

$$\dot{s} = -\rho\frac{s}{\psi} + \sum_{i=1}^{n} g_i(\epsilon_i + \tilde{\boldsymbol{W}}\boldsymbol{\beta}_i). \tag{43}$$

Eq.(43) has the structure of a low-pass filter with corner frequency $\rho\frac{s}{\psi}$ that shall be lower than 50Hz. This leads to a minimal value of the boundary thickness $\psi_{min} = 0.0003$. In the experiments, the thickness is much loosely chosen as $\psi = 0.2$.

The complete control law is

$$v = v_s + v_f \tag{44}$$

where v_f is the fast PD control for stabilising the joint deflection, which is selected according to an LQR design procedure offered by Quanser as

$$v_f = 43.7151(x_1 - x_3) + 0.511(\dot{x}_1 - \dot{x}_3). \tag{45}$$

The desired trajectory for the link angular position is a filtered square wave with initial position at 0.5rad. The desired and actual angular position of the link are presented in Figure 2 and the tracking error of the link in Figure 3. It is shown that the tracking error converges very fast to a range of approximately 0.05rad and remains there. Figure 4 shows the armature voltage of the DC motor, which is constrained to a reasonable range of less than 5V. Figure 5 represents the joint deflection, which is quite well damped by the fast PD controller. In further experiments (not presented here), a shorter beam is loosely mounted on the tip of the arm, which introduces a relatively strong time-variant uncertainty. Nevertheless, the system can maintain a quite decent tracking performance.

4. CONCLUSIONS

In this paper an adaptive sliding-mode control scheme was proposed. With the application of the backstepping design procedure, the requirement of the matching condition for classical sliding-mode control was removed. Gaussian radial-basis-function networks were used to approximate the unknown system dynamics. The stability analysis and the control law are simple. The controller shows very good performance with application to the tracking of a single-link flexible-joint manipulator.

Open questions are the choice of the radial-basis-function network type and its parameters, the application to more generic dynamic systems with other type of model uncertainties and also the extension to the MIMO case.

References

B. Drazenović. The invariance conditions in variable structure systems. *Automatica*, 5(3):287–295, 1969.

S. Gutman. Uncertain dynamical systems - A lyapunov min-max approach. *IEEE Transactions on Automatic Control*, 24(3):437–443, June 1979.

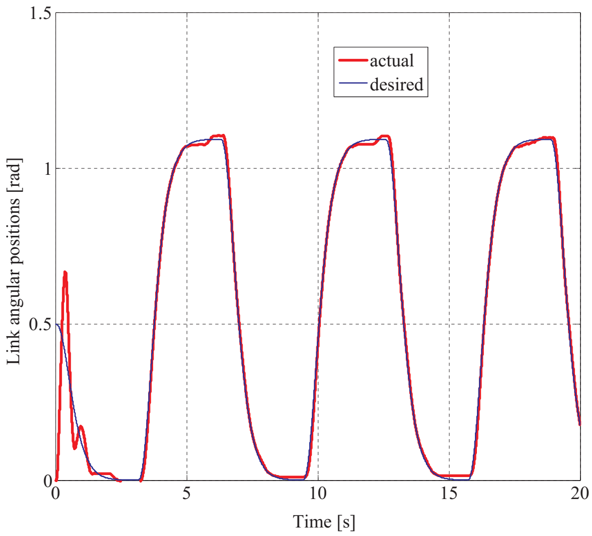

Fig. 2. Link angular position

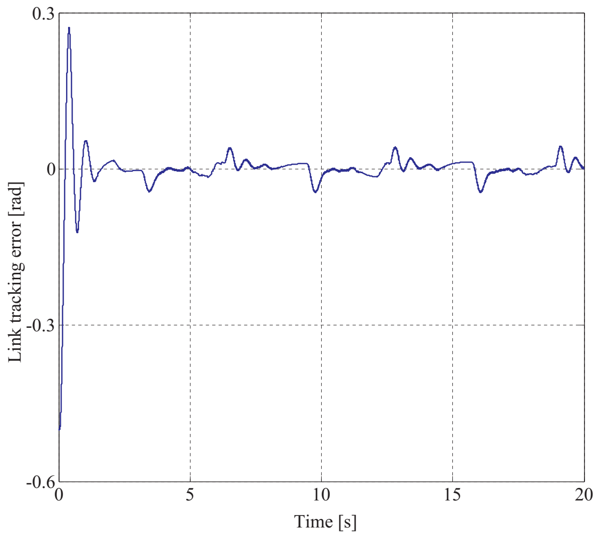

Fig. 3. Link tracking error

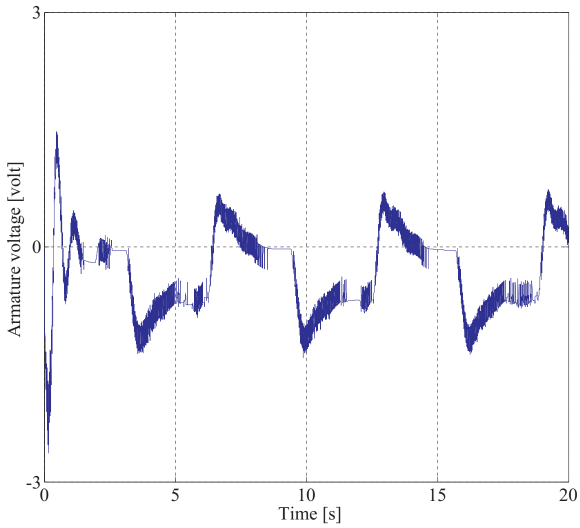

Fig. 4. Armature voltage of DC motor

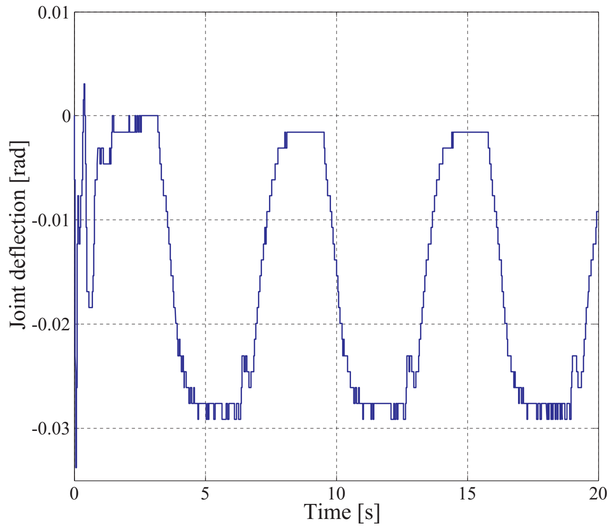

Fig. 5. Joint deflection

A.C. Huang and Y.C. Chen. Adaptive sliding control for single-link flexible-joint robot with mismatched uncertainties. *IEEE Transactions on Control Systems Technology*, 12(5):770–775, September 2004.

I. Kanellakopoulos, P.V. Kokotovic, and A.S. Morse. Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Transactions on Automatic Control*, 36(11): 1241–1253, 1991.

A.J. Koshkouei, R.E. Mills, and A.S.I. Zinober. Adaptive backstepping control. In X.H. Yu and J.X.Xu, editors, *Variable Structure Systems:Towards the 21st Century*, volume 274 of *Lecture Notes in control and Information Sciences*, pages 129–155. Springer, 2002.

L. Ma. *A Contribution to Adaptive Sliding–Mode Control with Gaussian Radial–Basis–Function Networks*. PhD thesis, Ruhr–Universität Bochum, 2005.

Quanser. *SRV02 Series Rotary Flexible Joint User Manual*. Quanser.

M. Rios-Bolivar, A.S.I. Zinober, and H. Sira-Ramirez. Dynamical adaptive sliding mode output tracking control of a class of nonlinear systems. *International Journal of Robust and Nonlinear Control*, 7:387–405, 1997.

R.M. Sanner and J-J.E. Slotine. Gaussian networks for direct adaptive control. *IEEE Transactions on Neural Networks*, 3(6):837–863, 1992.

J.J.E. Slotine and J.A. Coetsee. Adaptive sliding controller synthesis for non-linear systems. *International Journal of Control*, 43(6):1631–1651, 1986.

M.W. Spong. Control of flexible joint robots: a survey. In G. Chen, E.B. Lee, W. Littman, and L. Markus, editors, *New Trends and Applications of Distributed Parameter Systems*, Lecture Notes in Pure and Applied Mathematics, chapter 12, pages 355–383. Marcel Dekker, New York, 1990.

V.I. Utkin, J. Guldner, and J. Shi. *Sliding mode control in electromechanical systems*. Taylor & Francis, London, 1999.

K.K.D. Young and P.V. Kokotovic. Analysis of feedback loop interaction with actuator and sensor parasitics. *Automatica*, 18(5):577–582, 1982.