Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation, and Mathematics

PROCEEDINGS

17$^{\text{th}}$ International Conference on Process Control 2009

Hotel Baník, Štrbské Pleso, Slovakia, June 9 – 12, 2009

ISBN 978-80-227-3081-5

http://www.kirp.chtf.stuba.sk/pc09

Editors: M. Fikar and M. Kvasnica

Filasová, A., Krokavec, D.: Asymptotically Stable Control Design for Time-Delay Systems, Editors: Fikar, M., Kvasnica, M., In *Proceedings of the 17th International Conference on Process Control '09*, Štrbské Pleso, Slovakia, 358–362, 2009.

Full paper online: http://www.kirp.chtf.stuba.sk/pc09/data/abstracts/077.html

ASYMPTOTICALLY STABLE CONTROL DESIGN FOR TIME-DELAY SYSTEMS

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Abstract: The purpose of this paper is to present an improved version of time-delay system state feedback control methods and any extension over the one concerning the output and input variables constraint. Based on the standard Lyapunov-Krasovskii functional and norm-bounded constraints, delayed–independent stability condition is derived using linear matrix inequalities. The results obtained with a numerical example are presented to compare limitation in system structure for defined constraints. Since presented method is based on convex optimization techniques it is computationally very efficient.

Keywords: Constraints, linear matrix inequality, state feedback, time-delay systems, asymptotic stability.

1. INTRODUCTION

Continuous-time control systems are used in many industrial applications, where time delays can take a deleterious effect on both the stability and the dynamic performance in open and closed-loop systems. Therefore the stability and control of dynamical systems involving time-delayed states is a problem of large theoretical and practical interest where intensive activity are done to eliminate fixed time delays, to compensate for uncertain ones or to develop control for time-delay systems stabilization, especially for uncertain time-delay systems.

Number of techniques for time-delay linear systems control design as well as for stability analysis have been reported in the literature over past decades. Usually for the stability issue of time delay systems the Lyapunov–Krasovskii functional is used and results based on this functional are applied to controller synthesis and observer design. This time-delay independent methodology, as well as used bounded inequality techniques are sources of conservatism that can cause higher norm of state feedback gain (see e.g. Wang (2004)). Some progres review in this research field one can find in Gu et al. (2003), Niculescu at al. (2002), and the references therein.

This paper is concerned with the problem of asymptotically stable control design of continuous–time linear systems with delayed state, where the case of single, possibly varying time delay is considered and attention is focused on methods based on linear matrix inequalities (LMIs). Presented LMI approach is computationally efficient as it can be solved numerically using interior point methods (see e.g. Nesterov and Nemirovsky (1994)), and is based on norm bounded approximation for the Lyapunov–Krasovskii functional (see e.g. Kolmanovskii et al. (1999)), as well for new defined constrained extension of this functional.

2. PROBLEM DESCRIPTION

Through this paper the task is concerned with the computation of a state feedback $\boldsymbol{u}(t)$, which control a time-delay linear dynamic system given by the set of equations

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{A}_2\boldsymbol{q}(t-\tau) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{2}$$

with initial condition

$$\boldsymbol{q}(\vartheta) = \varphi(\vartheta),\ \forall\vartheta \in \langle -\tau, 0\rangle \tag{3}$$

where $\tau > 0$ is the state delay, $\boldsymbol{q}(t) \in I\!R^n$, $\boldsymbol{u}(t) \in \in I\!R^r$, and $\boldsymbol{y}(t) \in I\!R^m$ are vectors of the state, input and measurable output variables, respectively, and nominal system matrices $\boldsymbol{A} \in I\!R^{n\times n}$, $\boldsymbol{A}_2 \in I\!R^{n\times n}$, $\boldsymbol{B} \in I\!R^{n\times r}$ and $\boldsymbol{C} \in I\!R^{m\times n}$ are real matrices. Problem of the interest is to design asymptotically stable closed-loop system with the linear memoryless state feedback controller of the form

$$\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{q}(t) \tag{4}$$

Here matrix $\boldsymbol{K} \in I\!R^{r\times n}$ is the controller gain matrix.

It is supposed that the matrix

$$\boldsymbol{A}_{c2} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}) + \boldsymbol{A}_2 \tag{5}$$

has all its eigenvalues in the open left-half plane. The above assumption, which corresponds to the asymptotic stability of the closed-loop system without time delay, is indeed necessary for the global uniform asymptotic stability of closed-loop system in the presence of time delay.

3. BASIC PRELIMINARIES

3.1 *Schur Complement*

Nonlinear convex inequalities can be converted to LMI form using Schur's complements. Let a linear matrix inequality takes form

$$\begin{gathered}\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & -\boldsymbol{R} \end{bmatrix} < 0, \\ \boldsymbol{Q} = \boldsymbol{Q}^T,\ \boldsymbol{R} = \boldsymbol{R}^T,\ \det \boldsymbol{R} \neq 0 \end{gathered} \tag{6}$$

Using Gauss elimination, it yields

$$\begin{gathered}\begin{bmatrix} \boldsymbol{I} & \boldsymbol{S}\boldsymbol{R}^{-1} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & -\boldsymbol{R} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{I} \end{bmatrix} = \\ = \begin{bmatrix} \boldsymbol{Q} + \boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{R} \end{bmatrix}\end{gathered} \tag{7}$$

Since

$$\det \begin{bmatrix} \boldsymbol{I} & \boldsymbol{S}\boldsymbol{R}^{-1} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} = 1 \tag{8}$$

where $\boldsymbol{I}$ is the identity matrix of appropriate dimension, with this transform negativity of (6) is not changed, i.e. this follows as a consequence

$$\begin{gathered}\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & -\boldsymbol{R} \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} \boldsymbol{Q} + \boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{R} \end{bmatrix} < 0 \\ \Updownarrow \\ \boldsymbol{Q} + \boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T < 0, \quad \boldsymbol{R} > 0 \end{gathered} \tag{9}$$

respectively. As one can see, this complement offer possibility to rewrite nonlinear inequalities in a closed matrix LMI form (see e.g. Boyd at al. (1994), Krokavec and Filasová (2008)).

3.2 *Symmetric upper-bounds inequality*

Let $\boldsymbol{p} \in I\!R^n$, $\boldsymbol{r} \in I\!R^n$ are vectors of equal dimension. Then the next equality is satisfied

$$-\boldsymbol{p}^T\boldsymbol{r} - \boldsymbol{r}^T\boldsymbol{p} \leq \boldsymbol{p}^T\boldsymbol{X}^{-1}\boldsymbol{p} + \boldsymbol{r}^T\boldsymbol{X}\boldsymbol{r} \tag{10}$$

where $\boldsymbol{X} = \boldsymbol{X}^T > 0$, $\boldsymbol{X} \in I\!R^{n\times n}$, is any symmetric positive definite matrix. (see e.g. Li and de Souza (1997), Krokavec and Filasová (2007))

4. TIME–DELAY SYSTEM WITH OUTPUT CONSTRAINT

Defining the Lyapunov–Krasovskii functional with constraint as follows

$$\begin{gathered} v(\boldsymbol{q}(t)) = \\ = \boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{q}(t) + \int_{t-\tau}^{t} \boldsymbol{q}^T(r)\boldsymbol{R}\boldsymbol{q}(r)dr + \\ + \int_{t-\tau}^{t} \varepsilon\, \boldsymbol{y}^T(r)\boldsymbol{y}(r)dr > 0 \end{gathered} \tag{11}$$

where $\boldsymbol{P} = \boldsymbol{P}^T > 0$, and $\boldsymbol{R} = \boldsymbol{R}^T > 0$, $0 \leq \varepsilon < 1$, and evaluating derivative of $v(\boldsymbol{q}(t))$ one obtains

$$\begin{gathered} \dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t)\boldsymbol{P}\boldsymbol{q}(t) + \boldsymbol{q}^T(t)\boldsymbol{P}\dot{\boldsymbol{q}}(t) + \\ + \left(\boldsymbol{q}^T(t)\boldsymbol{R}\boldsymbol{q}(t) + \varepsilon\boldsymbol{y}^T(r)\boldsymbol{y}(r)\right)\Big|_{t-\tau}^{t} < 0 \end{gathered} \tag{12}$$

$$\begin{gathered} \dot{v}(\boldsymbol{q}(t)) = \\ = (\boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{A}_2\boldsymbol{q}(t-\tau) + \boldsymbol{B}\boldsymbol{u}(t))^T\boldsymbol{P}\boldsymbol{q}(t) + \\ + \boldsymbol{q}^T(t)\boldsymbol{P}(\boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{A}_2\boldsymbol{q}(t-\tau) + \boldsymbol{B}\boldsymbol{u}(t)) + \\ + \boldsymbol{q}^T(t)(\boldsymbol{R} + \varepsilon\boldsymbol{C}^T\boldsymbol{C})\boldsymbol{q}(t) - \\ - \boldsymbol{q}^T(t-\tau)(\boldsymbol{R} + \varepsilon\boldsymbol{C}^T\boldsymbol{C})\boldsymbol{q}(t-\tau) < 0 \end{gathered} \tag{13}$$

respectively. Using identity (10) with $\boldsymbol{X}=\boldsymbol{I}$ one can write

$$\begin{aligned}&\boldsymbol{q}^T(t-\tau)\boldsymbol{A}_2^T\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{A}_2\boldsymbol{q}(t-\tau)\le\\&\le\boldsymbol{q}^T(t-\tau)\boldsymbol{q}(t-\tau)+\boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{A}_2\boldsymbol{A}_2^T\boldsymbol{P}\boldsymbol{q}(t)\end{aligned}\tag{14}$$

and considering (14) it is possible to rewrite (13) in the form

$$\begin{aligned}&\dot{v}(\boldsymbol{q}(t))=\\&=\boldsymbol{q}^T(t)(\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{R})\boldsymbol{q}(t)+\\&+\boldsymbol{q}^T(t)(\boldsymbol{P}\boldsymbol{A}_2\boldsymbol{A}_2^T\boldsymbol{P}+\varepsilon\boldsymbol{C}^T\boldsymbol{C})\boldsymbol{q}(t)+\\&+\boldsymbol{q}^T(t-\tau)(\boldsymbol{I}-\boldsymbol{R}-\varepsilon\boldsymbol{C}^T\boldsymbol{C})\boldsymbol{q}(t-\tau)+\\&+\boldsymbol{u}^T(t)\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{B}\boldsymbol{u}(t)<0\end{aligned}\tag{15}$$

Then, with (4), inequality (15) implies

$$\begin{aligned}&\boldsymbol{q}^T(t)(\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{P}\boldsymbol{A}_2\boldsymbol{A}_2^T\boldsymbol{P}+\boldsymbol{R}+\\&+\varepsilon\boldsymbol{C}^T\boldsymbol{C}-\boldsymbol{K}^T\boldsymbol{B}^T\boldsymbol{P}-\boldsymbol{P}\boldsymbol{B}\boldsymbol{K})\boldsymbol{q}(t)+\\&+\boldsymbol{q}^T(t-\tau)(\boldsymbol{I}-\boldsymbol{R}-\varepsilon\boldsymbol{C}^T\boldsymbol{C})\boldsymbol{q}(t-\tau)<0\end{aligned}\tag{16}$$

$$\begin{bmatrix}\boldsymbol{q}^T(t) & \boldsymbol{q}^T(t-\tau)\end{bmatrix}\begin{bmatrix}\boldsymbol{\Phi}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{\Phi}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{q}(t)\\ \boldsymbol{q}(t-\tau)\end{bmatrix}<0\tag{17}$$

respectively, where

$$\begin{aligned}\boldsymbol{\Phi}_1&=\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{R}+\boldsymbol{P}\boldsymbol{A}_2\boldsymbol{A}_2^T\boldsymbol{P}+\\&+\varepsilon\boldsymbol{C}^T\boldsymbol{C}-\boldsymbol{K}^T\boldsymbol{B}^T\boldsymbol{P}-\boldsymbol{P}\boldsymbol{B}\boldsymbol{K}<0\end{aligned}\tag{18}$$

$$\boldsymbol{\Phi}_2=\boldsymbol{I}-\boldsymbol{R}-\varepsilon\boldsymbol{C}^T\boldsymbol{C}<0\tag{19}$$

Since $\boldsymbol{P}>0$, pre-multiplying (18) from the left side and right side by $\boldsymbol{P}^{-1}>0$ one can obtain

$$\begin{aligned}&\boldsymbol{\Psi}_1=\boldsymbol{P}^{-1}\boldsymbol{\Phi}_1\boldsymbol{P}^{-1}=\\&=\boldsymbol{P}^{-1}\boldsymbol{A}^T+\boldsymbol{A}\boldsymbol{P}^{-1}+\boldsymbol{A}_2\boldsymbol{A}_2^T-\boldsymbol{B}\boldsymbol{K}\boldsymbol{P}^{-1}-\\&-\boldsymbol{P}^{-1}\boldsymbol{K}^T\boldsymbol{B}^T+\boldsymbol{P}^{-1}(\boldsymbol{R}+\varepsilon\boldsymbol{C}^T\boldsymbol{C})\boldsymbol{P}^{-1}<0\end{aligned}\tag{20}$$

and setting

$$\boldsymbol{R}+\varepsilon\boldsymbol{C}^T\boldsymbol{C}=(1+\eta)\boldsymbol{I},\qquad\eta>0\tag{21}$$

$$\boldsymbol{Z}=\boldsymbol{K}\boldsymbol{P}^{-1},\qquad\boldsymbol{Y}=\boldsymbol{P}^{-1}\tag{22}$$

one can write for $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Phi}_2$

$$\begin{bmatrix}\boldsymbol{Y}\boldsymbol{A}^T+\boldsymbol{A}\boldsymbol{Y}+\boldsymbol{A}_2\boldsymbol{A}_2^T-\boldsymbol{B}\boldsymbol{Z}-\boldsymbol{Z}^T\boldsymbol{B}^T & \boldsymbol{Y}\\ \boldsymbol{Y} & -\delta\boldsymbol{I}\end{bmatrix}<0\tag{23}$$

$$\boldsymbol{\Phi}_2=-\eta\boldsymbol{I}=(1-\delta^{-1})\boldsymbol{I}<0\tag{24}$$

where $\delta\in I\!R$, $0<\delta=(1+\eta)^{-1}<1$, is a design parameter.

Especially, supposing $\eta=0$, i.e.

$$\boldsymbol{R}+\varepsilon\boldsymbol{C}^T\boldsymbol{C}=\boldsymbol{I}\tag{25}$$

one can obtain

$$\boldsymbol{\Phi}_2=\boldsymbol{0}\tag{26}$$

$$\begin{bmatrix}\boldsymbol{Y}\boldsymbol{A}^T+\boldsymbol{A}\boldsymbol{Y}+\boldsymbol{A}_2\boldsymbol{A}_2^T-\boldsymbol{B}\boldsymbol{Z}-\boldsymbol{Z}^T\boldsymbol{B}^T & \boldsymbol{Y}\\ \boldsymbol{Y} & -\boldsymbol{I}\end{bmatrix}<0\tag{27}$$

Conditions (26), (27) set this design task to be independent on system state time delay.

5. TIME–DELAY SYSTEM WITH OUTPUT AND INPUT CONSTRAINT

Generally it is possible to extend constraint in the Lyapunov–Krasovskii functional (11) as follows

$$v_e(\boldsymbol{q}(t))=v(\boldsymbol{q}(t))+\int_{t-\tau}^{t}\gamma\,\boldsymbol{u}^T(r)\boldsymbol{u}(r)dr>0\tag{28}$$

where $\varepsilon=1$, $\gamma>0$. Then derivative of Lyapunov–Krasovskii functional (28) takes form

$$\begin{aligned}&\dot{v}_e(\boldsymbol{q}(t))=\dot{v}\boldsymbol{q}(t))+\gamma\boldsymbol{u}^T(t)\boldsymbol{u}(t)-\\&-\gamma\,\boldsymbol{u}^T(t-\tau)\boldsymbol{u}(t-\tau)<0\end{aligned}\tag{29}$$

where $\dot{v}(\boldsymbol{q}(t))$ is given in (15). Then, with (4) and (15) inequality (29) implies

$$\begin{bmatrix}\boldsymbol{q}^T(t) & \boldsymbol{q}^T(t-\tau)\end{bmatrix}\begin{bmatrix}\boldsymbol{\Phi}_{e1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{\Phi}_{e2}\end{bmatrix}\begin{bmatrix}\boldsymbol{q}(t)\\ \boldsymbol{q}(t-\tau)\end{bmatrix}<0\tag{30}$$

where

$$\begin{aligned}\boldsymbol{\Phi}_{e1}&=\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+\boldsymbol{P}\boldsymbol{A}_2\boldsymbol{A}_2^T\boldsymbol{P}+\boldsymbol{R}+\\&+\boldsymbol{C}^T\boldsymbol{C}-\boldsymbol{K}^T\boldsymbol{B}^T\boldsymbol{P}-\boldsymbol{P}\boldsymbol{B}\boldsymbol{K}+\gamma\boldsymbol{K}^T\boldsymbol{K}<0\end{aligned}\tag{31}$$

$$\boldsymbol{\Phi}_{e2}=\boldsymbol{I}-\boldsymbol{R}-\boldsymbol{C}^T\boldsymbol{C}-\gamma\boldsymbol{K}^T\boldsymbol{K}<0\tag{32}$$

Therefore, one can write for $\boldsymbol{\Psi}_{e1}$

$$\begin{aligned}&\boldsymbol{\Psi}_{e1}=\boldsymbol{P}^{-1}\boldsymbol{\Phi}_{e1}\boldsymbol{P}^{-1}=\boldsymbol{P}^{-1}\boldsymbol{A}^T+\boldsymbol{A}\boldsymbol{P}^{-1}+\\&+\gamma\boldsymbol{P}^{-1}\boldsymbol{K}^T\boldsymbol{K}\boldsymbol{P}^{-1}+\boldsymbol{A}_2\boldsymbol{A}_2^T-\boldsymbol{B}\boldsymbol{K}\boldsymbol{P}^{-1}+\\&+\boldsymbol{P}^{-1}(\boldsymbol{R}+\boldsymbol{C}^T\boldsymbol{C})\boldsymbol{P}^{-1}-\boldsymbol{P}^{-1}\boldsymbol{K}^T\boldsymbol{B}^T<0\end{aligned}\tag{33}$$

and using (21), (22), to do it as follows

$$\begin{bmatrix}\boldsymbol{Y}\boldsymbol{A}^T+\boldsymbol{A}\boldsymbol{Y}+\boldsymbol{A}_2\boldsymbol{A}_2^T-\boldsymbol{B}\boldsymbol{Z}-\boldsymbol{Z}^T\boldsymbol{B}^T & \boldsymbol{Y} & \boldsymbol{Z}^T\\ \boldsymbol{Y} & -\delta\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{Z} & \boldsymbol{0} & -\gamma^{-1}\boldsymbol{I}_m\end{bmatrix}<0\tag{34}$$

Analogously it can be obtained

$$\begin{aligned}&\boldsymbol{\Psi}_{e2}=\boldsymbol{P}^{-1}\boldsymbol{\Phi}_{e2}\boldsymbol{P}^{-1}=\\&=-\eta\,\boldsymbol{P}^{-1}\boldsymbol{P}^{-1}-\gamma\,\boldsymbol{Z}^T\boldsymbol{Z}<0\end{aligned}\tag{35}$$

It is evident inequality (35) is negative definite since the identity matrix is positive definite matrix.

6. ILLUSTRATIVE EXAMPLE

A numerical example is provided below to illustrate main results. It is assumed that the parameters of a delay system (1), (2) are given by

$$\boldsymbol{A} = \begin{bmatrix} -7.36 & -2.76 & -13.80 \\ 19.56 & 8.96 & 31.80 \\ -5.68 & -3.88 & -5.40 \end{bmatrix}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -2.88 & -0.96 & -0.96 \\ 9.48 & 3.16 & 3.16 \\ -4.44 & -1.48 & -1.48 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.4 & -0.6 \\ -0.4 & 2.6 \\ 0.2 & 0.2 \end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}$$

and design parameters of Lyapunov–Krasovskii functional with output constraint (11) satisfies equality

$$\boldsymbol{R} + \varepsilon \boldsymbol{C}^T \boldsymbol{C} = \delta^{-1} \boldsymbol{I}, \qquad \delta = 0.9$$

Solving (23) for LMI matrix variables $\boldsymbol{Y}$ and $\boldsymbol{Z}$ using Self–Dual–Minimization (SeDuMi) package for Matlab (Peaucelle at al. (2002)), the feedback gain matrix design problem was solved as feasible with matrices

$$\boldsymbol{Y} = \begin{bmatrix} 3.1236 & -1.8626 & 2.9342 \\ -1.8626 & 5.9776 & 2.1982 \\ 2.9342 & 2.1982 & 6.3620 \end{bmatrix}$$

$$\boldsymbol{Z} = \begin{bmatrix} 14.0738 & 11.5758 & -92.5781 \\ 43.9268 & 72.1630 & 72.1595 \end{bmatrix}$$

Substituting $\boldsymbol{Y}$ and $\boldsymbol{Z}$ into (22) there was computed the feedback gain matrix as follows

$$\boldsymbol{K} = \boldsymbol{Z}\boldsymbol{Y}^{-1} = \begin{bmatrix} 437.3844 & 249.4492 & -302.4663 \\ 198.5989 & 118.5257 & -121.2061 \end{bmatrix}$$

One can easily verify, that closed loop is stable, with system matrices satisfying stability condition for

$$\boldsymbol{A}_c = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K} =$$
$$= \begin{bmatrix} -63.1544 & -31.4243 & 34.4629 \\ -321.8434 & -199.4270 & 225.9493 \\ -132.8767 & -77.4750 & 79.3345 \end{bmatrix}$$

$$\rho(\boldsymbol{A}_c) = \left\{ -8.5632 \ -13.9219 \ -160.7619 \right\}$$

$$\rho(\boldsymbol{A}_c + \boldsymbol{A}_2) =$$
$$= \left\{ -8.5572 \ -23.7091 \ -152.1807 \right\}$$

where $\rho(\cdot)$ denotes the eigenvalue spectrum of any matrix.

Using Lyapunov–Krasovskii functional (28), with output and input variable constraint parameters setting as $\varepsilon = 1$, $\gamma = 0.1$, there were no feasible solutions for LMI matrix variables $\boldsymbol{Y}$ and $\boldsymbol{Z}$ if matrix $\boldsymbol{A}_2$ is specified as above. Taking in computation another time-delay states system matrix $\boldsymbol{A}_2^\bullet$ chosen as follows

$$\boldsymbol{A}_2^\bullet = 0.2 \boldsymbol{A}_2$$

the problem was feasible with results

$$\boldsymbol{Y} = \begin{bmatrix} 0.5303 & -0.8413 & 0.0603 \\ -0.8413 & 1.6117 & 0.0815 \\ 0.0603 & 0.0815 & 0.2798 \end{bmatrix}$$

$$\boldsymbol{Z} = \begin{bmatrix} 0.4340 & 0.1380 & -1.0476 \\ 1.1634 & 2.6166 & 2.5129 \end{bmatrix}$$

$$\boldsymbol{K} = \begin{bmatrix} 17.0074 & 9.4774 & -10.1687 \\ 31.0407 & 17.9748 & -2.9407 \end{bmatrix}$$

$$\boldsymbol{A}_c = \begin{bmatrix} 4.4615 & 4.2339 & -11.4969 \\ -54.3430 & -33.9835 & 35.3782 \\ -15.2896 & -9.3704 & -2.7781 \end{bmatrix}$$

respectively. Since both eigenvalue spectrum of system matrices

$$\rho(\boldsymbol{A}_c) = \left\{ -5.1736 \ -10.7180 \ -16.4086 \right\}$$

$$\rho(\boldsymbol{A}_c + \boldsymbol{A}_2) = \left\{ -4.0292 \ -14.2555 \pm 3.7255\,\mathrm{i} \right\}$$

lie in the open left–half plain, designed control is stable.

7. CONCLUDING REMARKS

In this paper there was developed a constructive method based on a classical memoryless feedback control for the stabilization of time-delay systems with constraints given on output and input variables. The method ensure that the closed-loop system is internally stable in the sense of global uniform asymptotic stability in the presence of a state time delay. The validity of the proposed method is demonstrated by a numerical example with asymptotically stable closed-loop system variables.

ACKNOWLEDGMENTS

The work presented in this paper was supported by Grant Agency of Ministry of Education and Academy of Science of Slovak Republic VEGA under Grant No. 1/0328/08. This support is very gratefully acknowledged.

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